Characterization and Finite Element Modeling of Galfenol Minor Flux Density Loops

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ABSTRACT

This paper focuses on the development of a three-dimensional (3D) hysteretic Galfenol model which is implemented using the finite element method (FEM) in COMSOL Multiphysics. The model describes Galfenol responses and those of passive components including flux return path, coils and surrounding air. A key contribution of this work is that it lifts the limitations of symmetric geometry utilized in the previous literature and demonstrates the implementation of the approach for more complex systems than before. Unlike anhysteretic FEM models, the proposed model can simulate minor loops which are essential for both Galfenol sensor and actuator design. A group of stress-flux density loops for different bias currents is used to verify the accuracy of the model in the quasi-static regime.

Keywords: Galfenol, Finite Element Method, COMSOL Multiphysics, Hysteresis

1. INTRODUCTION

Magnetostrictive gallium-iron alloys, known as Galfenol, are a recent class of smart materials that are promising for sensing and actuation applications. To optimize the design of Galfenol sensors and actuators, system level models combining constitutive Galfenol modeling and passive system components are essential. Further, such models must be able to describe both minor and major magnetization excursions as this is the way Galfenol systems are often operated in practice.

Evans and Dapino proposed a high accuracy, computationally efficient model which uses energy averaging (EA) considering local energies near Galfenol’s six easy crystallographic directions. Evans derived a two dimensional (2D) static hysteretic finite element model for axisymmetric geometries. This model has been proved by Liang and Dapino for unimorph beam vibrations under a constant field assumption. Chakrabarti and Dapino simplified the EA model for axisymmetric 2D geometries such as those encountered in hydraulically-amplified Terfenol-D actuators. Although the model incorporates magnetic hysteresis, its application to general 3D geometries requires some additions. Chakrabarti and Dapino later formulated a 3D model for both static and dynamic operation, and implemented it in COMSOL Multiphysics. However, the model was formulated for anhysteretic responses in order to reduce computational complexity. It is emphasized that the response of Galfenol force sensors usually is in the form of minor loops, the nuances of which anhysteretic models cannot accurately describe.

To overcome these limitations, a 3-D hysteretic FEM model is presented in this paper. Fig. 1 shows both the geometry in COMSOL and the device developed for experimental validation of the model. The derivation process of weak forms, the modifications of the EA hysteretic Galfenol model, a model inversion procedure, and methods of improving convergence are presented in this paper. Quasi-static simulation results are compared with constant drive current measurements of stress-flux density minor loops on <100> oriented textured polycrystalline Fe$_{18.4}$Ga$_{81.6}$.

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2. THEORY

2.1 Weak form equations

The response of the Galfenol testing system in Fig. 1 can be described using two physics, magnetic domain and structural mechanics domain. The magnetic domain is formulated on the basis of Maxwell’s equations. To simplify the problem, displacement currents, Lorentz forces, and voltage gradients are neglected. With these assumptions, the weak form of magnetic physics can be written as

\[ \int_{V_B} \mathbf{H} \cdot \delta \mathbf{B} dV + \int_{\partial V_B} (\mathbf{H} \times \mathbf{n}) \cdot \delta \mathbf{A} d\partial V + \int_{V_B} \mathbf{J}_s \cdot \delta \mathbf{A} dV. \]  

(1)

The weak form equation for structural mechanics is derived from Newton’s law as

\[ \int_{V_u} \mathbf{T} \cdot \delta \mathbf{S} dV + \int_{V_u} (\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} + c \frac{\partial \mathbf{u}}{\partial t}) \cdot \delta \mathbf{u} dV + \int_{\partial V_u} \mathbf{t} \cdot \delta \mathbf{u} d\partial V + \int_{V_u} \mathbf{f}_B \cdot \delta \mathbf{u} dV. \]  

(2)

The independent variables are the vector magnetic potential \( \mathbf{A} \) and the mechanical displacement field \( \mathbf{u} \).

2.2 Hysteretic EA model

(a) Hysteretic Model for Galfenol

A complete derivation of anhysteretic Galfenol model has been shown by Chakrabarti and Dapino. Armstrong first proposed a constitutive model for Terfenol-D using energy weighting. Evans added stress-induced volume fraction change into the previous model, and successfully described the minor loops caused by stress variation. The volume fraction for hysteretic Galfenol responses is defined in an incremental form which includes both irreversible (hysteretic) and reversible (anhysteretic) volume fraction changes,

\[ d\xi^k = (1 - c)d\xi^k_{irr} + cd\xi^k_{an}, \]

(3)

where, \( d\xi^k \) is the volume fraction increment along the \( k^{th} \) moment orientation. Here, \( d\xi^k_{an} \) is the reversible volume fraction increment which can be written in analytical form. However, \( d\xi^k_{irr} \), the irreversible volume fraction is calculated numerically as shown by Evans and Dapino. The irreversible volume fraction maintains accuracy only when \( d\mathbf{H} \) and \( d\mathbf{T} \) are small enough,

\[ d\xi^k_{irr} = \frac{\zeta}{k_p}(\xi^k_{an} - \xi^k_{irr})[\mu_0 M_s(|dH_1| + |dH_2| + |dH_3|) + (3/2)\lambda_{100}(|dT_1| + |dT_2| + |dT_3|) + 3\lambda_{111}(|dT_4| + |dT_5| + |dT_6|)], \]

(4)

where \( \zeta \) depends on the sign of susceptibility, \( k_p \) quantifies pining site density in the material, and \( c \) describes the fraction of reversible moment rotations.
(b) Material Jacobian Evaluation
Based on the anhysteretic Jacobian presented by Chakrabarti and Dapino, a hysteretic Jacobian can be obtained through replacing the volume fraction term $\xi_{an}$ by $\xi$,

$$\frac{d\xi^k}{dH_i} = (1 - c)\frac{d\xi^{kr}}{dH_i} + c\frac{d\xi_{an}^k}{dH_i}, \quad i = 1, 2, 3,$$

$$\frac{d\xi^k}{dT_i} = (1 - c)\frac{d\xi^{kr}}{dT_i} + c\frac{d\xi_{an}^k}{dT_i}, \quad i = 1, 2, 3, 4, 5, 6. \quad (5)$$

Eq.(5) and Eq.(6) show that the derivatives are also defined in incremental form.

(c) Model Inversion
Fig. 2 illustrates the process of numerically approximating the weak form PDEs in COMSOL. It is noted that, unlike in the EA model, the independent variables are $(B, S)$ and the dependent variables are $(H, T)$. Hence, inversion of the EA model is necessary. It is not possible to derive analytical expressions for the inverse model from the EA model. A direct Newton method requiring calculation of the Jacobian matrix at each iteration step is not computationally efficient. Chakrabarti and Dapino applied the SR1 quasi-Newton method in the 3D anhysteretic model using nonlinear material functions. This method has proven to be both efficient and accurate, but lack of convergence is observed when the field or stress is large enough to drive the material to saturation.

![Figure 2. Flowchart of COMSOL simulation.](image)

Fig. 3 shows the flowchart of the SR1 quasi-Newton method applied in this study. To address the convergence issues, the following modifications are necessary. First, the initial conditions are set to be the model results in the previous step instead of all zeros. Second, values of $(H, T)$ and $(B, S)$ should be rescaled so that inputs and outputs of the inverse model are of the same order. After applying Eq.(7), where $SX$ and $SY$ are the scaling matrix for inputs and outputs respectively, model convergence is achieved. Third, a varying $\alpha$ is used such that it always satisfies the Wolfe condition. At last, Trust-Region-Reflective (TRR) method is required for special points where the quasi-Newton method converges to unphysical solutions.

$$SX = [1e-3; 1e-3; 1e-3; 1e-6; 1e-6; 1e-6; 1e-6; 1e-6; 1e-6];$$

$$SY = [1; 1; 1; 1e3; 1e3; 1e3; 1e3; 1e3; 1e3]. \quad (7)$$
2.3 Model Modifications

(a) Modified magnetization $M$ and magnetostriction $Sm$ equations

The bulk magnetization and magnetostriction have been expressed as weighted averages over the six crystal easy axes,

$$M_i = M_s \sum_{k=1}^{6} [m_i^{k} (\xi_i^{k} + d\xi_i^{k})],$$

$$Sm_i = \sum_{k=1}^{6} [S_i^{k} (\xi_i^{k} + d\xi_i^{k})].$$

(8)

All the components except $d\xi_i^{k}$ depend on previous step results. However, the magnetization $M_i m_i^{k}$ and magnetostriction $S_i^{k}$ along the $k^{th}$ moment orientation should be evaluated at current $H$ and $T$, in other words, the new equations should be written as:

$$M_i = M_s \sum_{k=1}^{6} [m_i^{k} (\xi_i^{k} + d\xi_i^{k})],$$

$$Sm_i = \sum_{k=1}^{6} [S_i^{k} (\xi_i^{k} + d\xi_i^{k})].$$

(9)

Eq.(8) requires small step size where the current $M_i m_i^{k}$ and $S_i^{k}$ differ slightly from stored values. However, if the previous weighted sum is applied to COMSOL, the inversion process actually returns $(dH, dT)$. Furthermore, $H_i = H_{i-1} + dH$ and $T_i = T_{i-1} + dT$, so the inversion error will accumulate leading to unphysical solutions. The inverse model designed on the basis of Eq.(9) does not purely depend on field and stress increment, so errors do not accumulate during simulation.

(b) Choice of coefficient $c$

According to the explanation in Section 2.3(a), the key for model convergence is to reduce the weight of incremental terms $(d\xi_{irr})$ in the constitutive model. The optimized $c$ value is usually between 0.1-0.2, which means over 80% of the $d\xi$ depends on $d\xi_{irr}$ and the majority of the material model is in incremental form. Hence, a small step size is required to maintain convergence. To reduce the influence of incremental terms, either increasing $c$ or decreasing $k_p$ is necessary. Hysteresis width is sensitive to the value of $k_p$, but $c$ has relative small influence on minor loop width when it is smaller than 0.5. A value $c = 0.4$ is selected which provides a good balance between efficiency and accuracy.
(c) Negative susceptibility
In previous hysteretic Galfenol models, $\zeta$ is forced to take a value of 1 when the calculated magnetic susceptibility is positive, otherwise it switches to 0. This 0 and 1 switching can cause simulation instabilities. One of the necessary conditions in the quasi-Newton method is that the function should be first order differentiable. In this study, $\zeta$ is always 1 to smooth the material model, and simulation results do not exhibit negative susceptibility.

(d) Anhysteretic and hysteretic model switch
Fig. 4 shows flux density versus stress responses for increasing bias fields. Two key characteristics are observed in the field-induced saturation region. First, $dB/dT$ is small, which means $T$ and $H$ are sensitive to the variation of $B$. A small perturbation of flux density causes large variation of field and stress, and the hysteretic EA model which is partially incremental cannot provide a converging solution. The other characteristic is that the anhysteresis curves almost coincide with the hysteretic ones at saturation. That is because all the moments have been aligned along the field direction, no moment rotation occurs, and hysteresis is negligible. This phenomenon makes it possible to replace the hysteretic model with the anhysteretic one in the saturation region. In this study, when $B > 1.37$ T, anhysteretic model automatically starts.

![Figure 4. Comparison of hysteretic experimental results and calculated anhysteretic curves.](image)

(e) Quasi-Newton method and Trust-Region-Reflective (TRR) method
In practice, the relationship between input ($H,T$) and output ($B,S$) is one to one reflection. In other words, only one group of ($H,T$) can be found for a given ($B,S$). However, the hysteretic EA model is accurate only when input variations are small between two continuous simulation steps. For example, if $c = 0.4$, the model should be limited to $|dH| <= 0.12$ kA/m and $|dT| <= 0.2$ MPa for a physical response. Outside of this range, unphysical values of ($H,T$) can generate a seemingly physical response. The converged solution of the quasi-Newton method only relies on the initial guess. Moreover, there are no convenient and reliable ways to select initial conditions such that the converged solution is guaranteed to be physically meaningful. Another numerical method that can search solutions within a given domain should be added. Matlab has a TRR method based function(fmincon). Each instance the SR1 method returns unphysical values for ($H,T$), the TRR method is used.

(f) Air gap and mesh quality
Initially, the air gap between the Galfenol rod and flux return path was ignored in order to reduce the degrees of freedom. However, this simplified model cannot converge when current drives the material near to the saturation state. Fig. 5 shows the flux density distribution near the contact region between the flux return path and the Galfenol rod. Without considering the air gap, flux concentrates in a small region of the rod, and this large flux density (maximum $|B| = 1.512$ T is...
observed in Fig. 5) is unrealistic and causes lack of convergence. Furthermore, the mesh quality is also a factor that influences convergence. In this study, a “normal” mesh is used for Galfenol, a “coarse” mesh is defined for passive domains.

3. RESULTS AND DISCUSSION

3.1 Simulation Results

Fig. 6 shows the flux density versus stress minor loops at different bias currents. Each hysteresis loop starts from a bias point which is calculated with the 3D anhysteretic model. The model results will gradually shift from anhysteretic bias points to the hysteretic minor loop in the first half cycle. After 1.5 cycles, the model converges to a complete and closed minor loop. It can be observed that the 3D hysteretic model can generally follow the trends of minor loop especially in the burst region. Fig. 7 shows field vs. stress calculations and data at different bias currents. It can be noticed that the T-H curve is more accurate when the stress is neither too small nor too large. This phenomenon will be explained in Section 3.2. The width of the hysteresis loop is correctly modeled and this validates the application of this model in the area where the energy loss is the main issue. This 3D model is initially designed to characterize stress-flux density curves and implemented to realize optimized design for Galfenol force sensors. Thus, the slope of the minor loop is the key factor that
should be used to quantify the simulation error. Linear regression is used to calculate the average slopes of all the minor loops (ignoring the initial half cycle). Both simulation and experimental results are shown in Table 1 and in Fig. 8.

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### 3.2 Error Analysis

(a) Parameter optimization error

Chakrabarti and Dapino\(^\text{10}\) have introduced the parameter optimization process for the anhysteretic EA model. Table 2 shows the parameters used in this study, and Fig. 9 compares the material model results with 1D measurements. The best choice for \(c\) is 0.152, but \(c = 0.4\) reduces computation time and maintains reasonable accuracy. Finally, the model exhibits 3.07% and 2.2% error for sensing and actuation, respectively.

(b) Permeability error

This COMSOL FEM model also simulates the current-magnetic field (i-H) relation using Maxwell’s equations. The modeling error for i-H is another error source. The permeability is related to the magnetic reluctance, which is the main factor that influences the i-H accuracy. By definition, the permeability of Galfenol, \(\mu_G = \frac{dB}{dH}\), the parameter fitting error would be amplified during
Figure 8. Sensitivity under different bias currents.

Table 2. Optimized Material Parameters

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<td>$G$</td>
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4. SUMMARY

This paper starts from a 3D anhysteretic FEM model. Several modifications are implemented to improve the convergence of the Chakrabarti and Dapino model in large stress and field inputs simulations. The energy averaged hysteresis model is extended to 3D FEM cases. Simplification and approximation methods are introduced to overcome the weakness of the incremental hysteresis model and simultaneously achieve computational efficiency. The model is validated using a $<100>$ oriented textured polycrystalline Fe$_{18.4}$Ga$_{81.6}$ rod in quasi-static condition. Comparison of simulations and experiments proves that the 3D hysteretic model is accurate over various bias current and input stress ranges. Based on this quasi-static hysteretic FEM modeling, a dynamic model for minor loop response will be built. Frequency-dependent stress-flux density minor loops measured by Walker et al. are available to validate the future 3D hysteretic dynamic FEA model.
Figure 9. Parameter optimization results. Upper: Flux density vs. field at different bias stresses; Lower: Flux density vs. stress at different bias field.
Figure 10. Parameter optimization results of permeability.

Figure 11. Hysteresis width with respect to different $c$ values.
Currently, this FEA framework requires that the easy direction of Galfenol aligns along the input stress and field direction. A coordinate transformation strategy can be added to extend this model to arbitrary input directions.

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