Modeling Major and Minor Hysteresis Loops In Galfenol-Driven Micro-Positioning Actuators Using A Memory-Based Hysteresis Framework

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Abstract: This article presents a memory-based hysteresis modeling framework which uses a mathematical mapping technique for accurate prediction of major and minor hysteresis loops in Galfenol-driven actuators. The model is based on three properties of hysteretic materials which have recently been established for piezoelectric actuators. These properties are: targeting of turning points, curve alignment and the wiping-out effect. To describe the hysteretic behavior of the actuator, we initially separate the nonlinearity of the anhysteretic response from the hysteresis looping effect and approximate it with a piecewise exponential function. This function is then utilized in a nonlinear mapping procedure, where it is mapped between consequent turning points recorded in the model’s memory unit. This mapping requires two constant shaping parameters for the ascending and two for the descending trajectories. To assess the performance of the proposed model and for experimental verification, a Galfenol-driven micro-positioning cantilever actuator is . The results indicate that the model is able to precisely predict the response of the actuator for its full-range motion including both major and minor hysteresis loops. The maximum error percentage and the average error between the experiment and the model for a 120 µm stroke are 1.5% and 0.4364 µm, respectively. With using only 14 coefficients, the proposed framework not only offers a precise model, but also a computationally efficient algorithm to quantify the hysteresis response of Galfenol-driven actuators.

1. INTRODUCTION
Magnetostriction (magnetically-induced strain) and its complementary effect, stress-induced magnetization, originate from the coupling between interatomic spacing and magnetic moment orientation 1. Magnetostriction is thus a material property which does not degrade with time or repetition of thermo-magneto-mechanical cycles. Magnetostrictive materials have facilitated dynamic actuators and sensors with robust operation, unlike ferroelectric materials, and broad frequency bandwidth, unlike shape memory materials. Magnetostrictive Galfenol possesses key additional advantages; unlike most active materials, Galfenol is malleable and machinable 2 and can be safely operated under simultaneous tension, compression, bending, and shock loads. As a consequence of the unique combination of metallurgical and mechanical properties of Galfenol, this material can enable smart load-carrying Galfenol devices and structures with innovative 3-D manufactured by welding, extrusion, rolling, deposition, or machining. Furthermore, the unprecedented control of anisotropies through manufacturing and post-processing methods made possible with Galfenol 3 can lead to innovative devices with fully coupled 3-D functionality 4.

Despite these advantages, Galfenol does exhibit magnetic saturation, magnetic hysteresis and magnetomechanical nonlinearities, and accurate models describing the effect of magnetic field on strain are necessary for device design and control. A set of descriptive memory-based properties has recently been proposed as providing sufficient knowledge for precise prediction of major and minor hysteresis loops. This approach has been validated for hysteresis nonlinearity in piezoelectric actuators 5-7. It has been shown that for precise prediction of hysteresis, a number of memory units must be included in the model to record key points of past hysteresis trajectories and use them for prediction of future paths. The proposed model has been shown to offer better accuracy and improved computational efficiency compared to other existing methods.

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Building on the memory-based approach, a generalized framework is proposed in this article for modeling the highly nonlinear multiple-loop hysteresis behavior of Galfenol micro-positioning actuators. We separate the nonlinear anhysteretic response from the hysteresis looping effect, and approximate it with an exponential average function. Using the memory-based properties of hysteresis, a formulation is then derived in which the average function is incorporated in development of hysteresis loops. The proposed model is experimentally identified and validated for a Galfenol actuator. Results indicate good agreement of the model response with the experimental data in predicting both major and minor loops. The advantages of the model over the existing methods include computational and parametric efficiency, simplicity of identification, and accuracy of response.

Fig 1. Experimental setup for a Galfenol-driven micro-positioning actuator.

2. EXPERIMENTAL ANALYSIS

To investigate the memory-based behavior of Galfenol actuators, an experimental setup comprising a Galfenol-driven actuator was developed as shown in Figure 1. A unimorph beam consisting of a Galfenol laminate bonded to a brass laminate was placed in a magnetic circuit composed of steel flux paths driven by a copper coil. Both laminates have dimensions $1'' \times 0.25'' \times 0.015''$ and were bonded with Armstrong A12 adhesive. The Galfenol is research grade Fe$_{81.6}$Ga$_{18.4}$ which has been stress-annealed in order to pre-align magnetic domains perpendicular to the length of the beam thereby providing optimal magnetostriction (~200 ppm) along the beam length. When the coil is energized, a magnetic field causes the Galfenol to elongate while the brass laminate causes it to bend by restricting its elongation along the contact face. The coil is driven by an AE Techron LVC 5050 Linear Amplifier operated in voltage control mode. Current measurements are provided by the amplifier at a gain of 20 A/V and displacements are measured by a linear variable differential transformer (LVDT) instrument having a gain of 0.0124742 in/V. The LVDT instrument comprises a Lucas Schaevitz MHR025 sensor and ATA-101 amplifier. A SignalCalc ACE dynamic signal analyzer from Data Physics Corporation simultaneously controls the drive coil amplifier and acquires the displacement and current measurements.

Major and nested minor loop tests were conducted using ramp inputs with a rate of 6 V/sec and sampling frequency of 40 Hz. Closure of the minor loops suggests that dynamic effects are negligible at the frequency tested.

Fig 2. Nonlinear hysteretic response of Galfenol actuator: (a) Input current profile, (b) actuator displacement, and (c) butterfly-like hysteresis loops.

Figure 2 depicts the response of the actuator to an input current signal with a triangular profile. As seen in Figure 2(b)-(c), the response is nonlinear with a butterfly-shaped configuration and the looping effect due to magnetic hysteresis. While the saturation nonlinearity dominates over the hysteresis, precision requires that both be taken into account. From Figure 2(c) which shows the hysteresis loops between the applied current and the resulting displacement, two important characteristics are observed which will be applied to the modeling framework: (i) the hysteresis loops are symmetric with respect to the vertical axis, and (ii) the center of the loops, where the ascending and descending curves cross each other, is located at the origin. Therefore, the butterfly configuration can be reduced to a single-sided
hysteresis configuration without the loss of generality and for
the sake of simplicity.

3. MODELING NONLINEARITIES USING AVERAGE CURVE MODEL

In this section, an exponential average curve fitting procedure
is used to identify the best fit for the combined ascending and
descending hysteresis curves. This method only predicts the
nonlinearity of the response without considering its hysteresis
looping effect. However, the obtained average function can
be directly utilized in a generalized memory-based hysteresis
modeling framework, as discussed in section 4.

3.1. Average curve model:
The nonlinear anhysteretic response is obtained from the
average curve between the ascending and descending
reference curves as shown in Figure 3(a). Therefore, the
model is simply reduced to a single function between the
applied current and the resulting displacement. The following
exponential formulation precisely approximates the
anhysteretic average curve:

\[
x = f(v) = \sum_{i=1}^{m} a_i v^{i-1} \left[ 1 - \exp \left( \sum_{j=1}^{n} b_j v^j \right) \right]
\]  

(1)

where \( v \) and \( x \) respectively represent the input current and
output displacement, and \( a_i \) and \( b_j \) are constant shaping
coefficients. Choosing the appropriate polynomial orders \( m \)
and \( n \), a least square optimization can be used to identify the
unknown shaping parameters based on a set of given
experimental data.

3.2. Experimental results and numerical simulations:
Letting \( n = m = 5 \) in (1) to precisely approximate the
experimental data, the following expression is obtained for
the average function:

\[
x = f(v) = \left( a_0 + a_1 v + a_2 v^2 + a_3 v^3 + a_4 v^4 \right)
\times \left[ 1 - e^{ \left( b_1 v + b_2 v^2 + b_3 v^3 + b_4 v^4 + b_5 v^5 \right) } \right]
\]  

(2)

The response of the average model with the input shown in
Figure 2(a) is compared to the experimental response in
Figure 3. The maximum error percentage and the average
error are 5.0232% and 1.7602 µm, respectively. The induced
error is due to the multi-path hysteretic response of the
actuator; hence, a precise hysteresis model must be
incorporated for acquiring higher precision, as will be
discussed in the next section.

4. MEMORY-BASED HYSTERESIS MODELING

The memory-based hysteresis modeling framework, first
developed for piezoelectric actuators 5-7, is proposed here to
investigate the modeling feasibility of the Galfenol hysteretic
behavior. This framework is based on three important
properties: targeting of turning points, curve alignment and
the wiping out effect, which enable precise prediction of the
multiple-loop hysteresis response of active materials. To
develop the memory-based hysteresis model for a Galfenol
actuator, the nonlinearity of the anhysteretic response is
separated from its hysteretic looping behavior. Figure 4(a)
shows that ascending and descending hysteresis trajectories
can share the nonlinearity from a single average curve which
passes through them. However, to include the hysteresis
effect, a mathematical mapping is utilized to split the average
curve into appropriate ascending and descending trajectories.
Before representing a mathematical formulation, a summary
of hysteresis properties is explained in the following
paragraph.

Starting from the origin, the initial hysteresis trajectory, when
moving upward, targets the upper threshold point corresponding to the maximum possible input. Likewise, the
first descending trajectory targets the lower threshold point corresponding to the minimum allowed input. When the
direction of the input changes, the information of that turning
point is recorded to be utilized for the prediction of future
hysteresis paths. A hysteresis trajectory starting from a
turning point approaches the closest recorded turning point
on the opposite side. This property, termed targeting of the
turning points, enables the prediction of the hysteresis path
between two subsequent turning points. When the trajectory
hits a target point, the minor hysteresis loop associated with that point is closed and the properties of wiping-out and curve alignment occur simultaneously. The wiping-out effect implies that when a minor loop is closed, it is no longer useful for the prediction of future hysteresis paths. Therefore, the information of that loop and its associated turning points can be eliminated from the memory unit. This property is advantageous as it helps to vacate the memory units from unnecessary information and keep them for recording the forthcoming turning points. Another advantage of the proposed framework is the curve alignment property observed at the turning points; that is the hysteresis trajectory changes path slightly and aligns to the previously broken curve associated with that turning point, and continues its path towards the next target point.

4.1 Mathematical formulation:
To mathematically model a nonlinear and hysteretic response, we first focus on predicting a hysteresis track between a starting turning point \((v_1, x_1)\) and its target point \((v_2, x_2)\). The following nonlinear mapping is proposed for this hysteresis track:

\[
x_{12}(v) = x(v; v_1, x_1, v_2, x_2) = x_1 + \left\{ k \left( 1 + ae^{-r[(f(v) - f(v_1))]}\right) \frac{1}{(f(v) - f(v_1))} \right\}
\]

where \(v\) represents the operator input (applied current in the case of a Galfenol actuator), \(a\) and \(\tau\) are the mapping coefficients being separately identified for the ascending and descending curves, and \(f\) stands for the average function given by (1). Coefficient \(k\) represents the relative slope of mapping and is expressed as:

\[
k = \frac{x_2 - x_1}{(f(v_2) - f(v_1))} \left( 1 + ae^{-r[(f(v) - f(v_1))]\right)}
\]

Substitution of (4) into (3) for the hysteresis track between point \((v_1, x_1)\) and \((v_2, x_2)\) yields:

\[
x_{12}(v) = x_1 + \left\{ \left( 1 + ae^{-r[(f(v) - f(v_1))]\right)} \frac{1}{(f(v_2) - f(v_1))} \right\} (x_2 - x_1)
\]

The fact that the hysteresis trajectory starting from a turning point targets all the previously recorded internal turning points enables the prediction of the response utilizing the proposed mapping strategy. Figure 4(b) depicts a typical hysteresis path originating from point \((v_0, x_0)\). If the input keeps increasing up to the maximum threshold point, the trajectory passes through all the internal target points \((v_1, x_1), (v_2, x_2), \ldots, (v_{n+1}, x_{n+1})\) and approaches the upper threshold point \((v_x, x_a)\). The equation describing this path is given by:

\[
x_{12}(v) = \sum_{i=0}^{n} H(v, v_i, v_{i+1}) \times 
\left\{ \left( 1 + ae^{-r[(f(v) - f(v_i))]\right)} \frac{1}{(f(v) - f(v_i))} \right\} (x_{i+1} - x_i)
\]

where \(x_{12}(v)\) denotes the predicted multi-segment hysteresis path, which will hold until the direction of input changes, and \(H\) represents the unit bilateral Heaviside function expressed as:

\[
H(x, a, b) = \begin{cases} 
1 & a \leq x \leq b \\
0 & x > b \text{ or } x < a
\end{cases}
\]

By dividing the model into ascending and descending parts, identifying the average function parameters and mapping coefficients, assigning a memory organizer to record and wipe out the turning points, and initializing the memory units to the upper and lower threshold points, a model can be developed based on (6) to predict any hysteresis response occurring between the lower and the upper threshold points.

Figure 4. (a) Separation of nonlinearity from the hysteresis loops, and (b) typical multi-segment internal path.

4.2. Experimental verification
A least square error minimization algorithm has been utilized to identify parameters \(a\) and \(\tau\) for the ascending and descending reference curves. Once these parameters are identified, they remain unchanged for the internal trajectories. This comes from the fact that all the internal hysteresis trajectories adopt their shape from the associated reference curves. The identified memory-based model is subjected to the same input profile given in Figure 2(a). The response of the model is compared to experimental data in Figure 5. As seen, the butterfly hysteresis loops have been precisely predicted through the proposed model. Furthermore, Figure 6 depicts the effectiveness of the model in predicting minor hysteresis loops. It is emphasized that to force the average curve to fit the ascending and descending reference curves only 4 parameters have been used (2 for ascending and 2 for descending curves). Adding the 10 coefficients of the average function, a total of 14 parameters are required to model the nonlinear behavior of a Galfenol micro-positioning actuator.
To investigate the performance improvement of the proposed memory-based hysteresis framework over the average curve model, error signals of both models are compared in Figure 7. Maximum error percentages and the average error values are presented in Table 1. The results indicate that around 400% improvement is achieved when the hysteresis prediction is added to the average curve model. For further demonstration of the actuator response to arbitrary input profiles, a numerical simulation is carried out as depicted in Figure 8. Results demonstrate the ability of the model in predicting multiple-loop hysteresis response for negative and positive values of input and for both left and right minor loops.

It is remarked that only rate-independent hysteresis loops can be modeled through this framework. To compensate for the effects of system frequency-dependent dynamics, appropriate models must be selected to account for the dynamical effects including system inertia and damping.

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<th>Table 1. Comparison of the error values for the average curve model and the memory-based hysteresis model.</th>
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<td>Model</td>
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<td>Average curve model</td>
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<td>Memory-based model</td>
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5. CONCLUSIONS

A memory-based hysteresis modeling framework has been proposed to predict the nonlinear hysteresis behavior of Galfenol micro-positioning actuators. First, the nonlinearity of the response is separated from its hysteresis looping effect and an average exponential formulation is used to eliminate the major source of inaccuracy due to the nonlinearity. Using the memory-based properties of hysteresis, a generalized memory-based formulation is then derived for prediction of hysteresis loops in the Galfenol actuator. Results indicate improvements of model performance by up to 400% over the average function model. Moreover, with using only 14 coefficients for prediction of the nonlinear and hysteretic displacement vs. current of a Galfenol actuator, the model can be considered as a computationally efficient approach compared to the present methods. The proposed model can be applied to a variety of smart materials for model-based feedforward and feedback control schemes.
Figure 8. Hysteresis model performance: (a) input profile, (b) butterfly hysteresis response, (c) left minor loops and (d) right minor loops.

6. ACKNOWLEDGEMENT

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7. REFERENCES


