Magnetomechanical characterization and unified actuator/sensor modeling of ferromagnetic shape memory alloy Ni-Mn-Ga

Neelesh N. Sarawate\textsuperscript{a}, Marcelo J. Dapino\textsuperscript{a}

\textsuperscript{a} Department of Mechanical Engineering, The Ohio State University, Columbus, OH, USA, 43210

ABSTRACT

A unified thermodynamic model is presented which describes the bulk magnetomechanical behavior of single-crystal ferromagnetic shape memory Ni-Mn-Ga. The model is based on the continuum thermodynamics approach, where the constitutive equations are obtained by restricting the thermodynamic process through the Clausius-Duhem inequality. The total thermodynamic potential consists of magnetic and mechanical energy contributions. The magnetic energy consists of Zeeman, magnetostatic, and anisotropy energy contributions. The microstructure of Ni-Mn-Ga is included in the continuum thermodynamic framework through the internal state variables domain fraction, magnetization rotation angle, and variant volume fraction. The model quantifies the following behaviors: (i) stress and magnetization dependence on strain (sensing effect), and (ii) strain and magnetization dependence on field (actuation effect).

1. INTRODUCTION

Ferromagnetic shape memory alloys (FSMAs) in the Ni-Mn-Ga system are a class of magnetically-activated smart material. The single crystal Ni-Mn-Ga typically exhibits 6% strains in the presence of magnetic fields of around 700 kA/m.\textsuperscript{1} Due to the magnetic field activation, they also exhibit higher bandwidth compared to conventional shape memory alloys.\textsuperscript{2} The large strain and broad frequency bandwidth provide a desirable operating space for actuator applications. Several models have been developed to describe the actuation effect of single-crystal Ni-Mn-Ga.

Our prior experimental measurements demonstrate the sensing potential of Ni-Mn-Ga.\textsuperscript{3} A continuum thermodynamics model was developed which describes these experimental results.\textsuperscript{4} Our model is constructed with magnetization and strain as the independent variables, unlike similar models by Hirsinger,\textsuperscript{5} Kiefer and Lagoudas\textsuperscript{6} and Faidley,\textsuperscript{7} which are based on the actuation effect and thus utilize magnetic field and stress as the independent variables. Further, our model includes the magnetostatic energy and demagnetization effects.

In our sensing model, a special form of the Gibbs free energy including magnetic and mechanical contributions is used as a thermodynamic potential. To incorporate the microstructure of Ni-Mn-Ga in the continuum thermodynamic framework, the following internal state variables are considered: domain fraction, magnetization rotation angle, and variant volume fraction. The constitutive equations for stress and magnetization are obtained by restricting the process through the Clausius-Duhem inequality (second law of thermodynamics). The model parameters used in the model can be obtained from two simple tests.

In this paper, the same sensing model presented by Sarawate and Dapino\textsuperscript{4} is utilized to characterize Ni-Mn-Ga actuation. An important feature of our unified model is that the actuation and sensing are characterized by the exact same set of six parameters. The actuation model describes the dependence of strain and magnetization on the external magnetic field. The Gibbs energy is used as thermodynamic potential with field and stress as independent variables. The experimental results presented by Murray\textsuperscript{8} are used to verify the model performance.

The sensing model is described in section 2; this section presents discussions on the thermodynamic framework considering elastic and magnetic terms, contributing energy terms, evolution of internal state variables, and

Further author information: (Send correspondence to M.J.D)

N.N.S.: E-mail: sarawate.1@osu.edu, Telephone: 1-614-247-7480

M.J.D.: E-mail: dapino.1@osu.edu, Telephone: 1-614-688-3689


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results. Section 3 describes the extension of the model to the actuation effect and presents a comparison of model results with experimental data.

2. CONTINUUM THERMODYNAMIC MODEL FOR THE SENSING EFFECT

2.1. Thermodynamic Framework

For a magneto-mechanical material undergoing an isothermal process, the Clausius-Duhem inequality that describes the second law of thermodynamics has the form

\[-\rho \dot{\psi} + \sigma \dot{\varepsilon} + \mu_0 H \dot{M} \geq 0,\]  

(1)

where \(\psi\) is the thermodynamic potential in the form of Helmholtz energy, \(H\) is the externally applied magnetic field, \(M\) is the magnitude of magnetization in the direction of applied field, \(\sigma\) is the engineering stress along the longitudinal direction of the sample, and \(\varepsilon\) is the strain. This material has constitutive dependencies

\[\psi = \psi(\varepsilon, M)\]
\[\sigma = \sigma(\varepsilon, M)\]
\[H = H(\varepsilon, M).\]  

(2)

For the sensing effect under consideration, the strain is applied externally; hence, it is desirable to have strain as an independent variable. A thermodynamic potential termed magnetic Gibbs energy is therefore obtained through Legendre transformation

\[\rho \phi = \rho \psi - \mu_0 H M.\]  

(3)

Substitution of (3) into (1) results in a modified Clausius-Duhem inequality

\[-\rho \dot{\phi} + \sigma \dot{\varepsilon} - \mu_0 M \dot{H} \geq 0\]  

(4)

which describes a magneto-mechanical material with the following constitutive dependencies,

\[\phi = \phi(\varepsilon, H, \alpha, \theta, \xi)\]
\[\sigma = \sigma(\varepsilon, H, \alpha, \theta, \xi)\]
\[M = M(\varepsilon, H, \alpha, \theta, \xi).\]  

(5)

Here, the internal state variables \(\alpha\), \(\theta\), and \(\xi\) denote the domain fraction, magnetization angle, and variant volume fraction. The internal state variables are generally used to include the microstructure and dissipative effects in a continuum model.

A simplified two-variant Ni-Mn-Ga microstructure is shown in Figure 1, where the applied field is oriented in the \(x\) direction and the applied stress (or strain) is oriented in the \(y\) direction. A field-preferred variant is one with its \(c\)-axis aligned with the \(x\) direction and volume fraction \(\xi\). A stress-preferred variant has its \(c\)-axis aligned with the \(y\) direction and volume fraction \(1 - \xi\). Each variant consists of a collection of 180-degree domains which are formed in order to minimize the net magnetostatic energy due to finite dimensions of the sample. Neighboring domains have magnetization vectors oriented opposite to each other, with volume fraction \(\alpha\) and \(1 - \alpha\).

In the absence of an external field, the domain fraction \(\alpha = 1/2\) leads to minimum magnetostatic energy. Due to the high magnetocrystalline anisotropy energy of Ni-Mn-Ga, the magnetization vectors tend to attach to the crystallographic \(c\)-axis. Any rotation of the magnetization vectors away from the \(c\)-axis thus results in an increase in the anisotropy energy. The magnetization vectors in the field-preferred variants are always attached to the \(c\)-axis of the crystals, i.e., they are aligned with the applied field. However, the direction of the magnetization can be either in the direction of the field, or opposing it. The magnetization vectors in a stress-preferred variant are rotated at an angle \(\theta\) relative to the \(c\)-axis. This angle is equal and opposite in the two magnetic domains within a stress-preferred variant.

Using the chain rule, equation (4) becomes

\[\sigma - \frac{\partial \phi}{\partial \varepsilon} \dot{\varepsilon} + (-\mu_0 M - \frac{\partial \phi}{\partial H}) \dot{H} + \pi^\alpha \dot{\alpha} + \pi^\theta \dot{\theta} + \pi^\xi \dot{\xi} \geq 0,\]  

(6)
where the terms $\pi^\alpha = -\partial(\rho\phi)/\partial\alpha$, $\pi^\theta = -\partial(\rho\phi)/\partial\theta$, and $\pi^\xi = -\partial(\rho\phi)/\partial\xi$ represent thermodynamic driving forces respectively associated with internal state variables $\alpha$, $\theta$, and $\xi$. In equation (6), the terms $\dot{\varepsilon}$ and $\dot{H}$ are assumed to be independent of each other, and of other rates. Thus, for an arbitrary process, the coefficients of $\varepsilon$ and $H$ must vanish in order for the inequality to hold. This leads to the constitutive equations

$$\sigma = \frac{\partial(\rho\phi)}{\partial \varepsilon}$$  \hspace{1cm} (7)$$

$$M = -\frac{1}{\mu_0} \frac{\partial(\rho\phi)}{\partial H}.$$  \hspace{1cm} (8)$$

Thus, the Clausius Duhem inequality is reduced to

$$\pi^\alpha \dot{\alpha} + \pi^\theta \dot{\theta} + \pi^\xi \dot{\xi} \geq 0.$$  \hspace{1cm} (9)$$

By inspection of Figure 1, the magnetization in the $x$ direction can be written as

$$M(\xi, \alpha, \theta) = M_s[2\xi\alpha - \xi + \sin \theta - \xi \sin \theta].$$  \hspace{1cm} (10)$$

2.2. Energy Formulation

2.2.1. Magnetic energy

The total magnetic potential energy of a Ni-Mn-Ga sensor/actuator includes Zeeman, magnetostatic, and magnetocrystalline anisotropy contributions. The Zeeman energy, which represents the energy available to drive the twin boundary motion by external magnetic fields, is minimum when the magnetization vectors are completely aligned in the direction of the externally applied field, and is maximum when the magnetization vectors are aligned in the direction opposite to the field. For the simplified two-variant system shown in Figure 1 with net magnetization component $M$ in the direction of externally applied field $H$, the Zeeman energy has the form

$$\rho\phi_{ze}(H, \alpha, \theta, \xi) = -\mu_0 M_s(2\xi\alpha - \xi + \sin \theta - \xi \sin \theta)H.$$  \hspace{1cm} (11)$$

The magnetostatic energy represents the energy opposing the external work done by magnetic fields, on account of the geometry of the specimen. The magnetization creates a demagnetization field which opposes the externally applied field and whose strength depends on the geometry and permeability of the material. For the system shown in Figure 1, the magnetostatic energy is given by

$$\rho\phi_{ms}(\xi, \alpha, \theta) = \frac{1}{2}\mu_0 NM_s^2(2\xi\alpha - \xi + \sin \theta - \xi \sin \theta)^2.$$  \hspace{1cm} (12)$$
where $N$ represents the difference in the magnetization factors along the $x$ and $y$ directions.\textsuperscript{10}

The anisotropy energy represents the energy needed to rotate a magnetization vector away from its corresponding easy axis. This energy is minimum (or zero) when the magnetization vectors are aligned along the $c$-axis and is maximum when they are rotated 90 degrees away from the $c$-axis. In Figure 1, the only contribution to the anisotropy energy is from the stress preferred variant. The anisotropy energy is thus given by

$$\rho \phi_{an}(\alpha, \theta, \xi) = K_u (1 - \xi) \sin^2 \theta.$$  \hfill (13)

The anisotropy constant, $K_u$, is calculated from the experimental M-H curves as the difference in the area under the easy and hard axis M-H curves. It represents the energy associated with pure rotation of the magnetization vectors (hard axis) compared to the magnetization due to zero rotation of vectors (easy axis). The anisotropy constant of the material employed for model validation is $1.675 \times 10^5 \text{ J/m}^3$.

The total magnetic Gibbs energy can be written as the sum of Zeeman, magnetostatic, and anisotropy energy,

$$\rho \phi_{mag}(H, \xi, \alpha, \theta) = -\mu_0 M_s (2\xi \alpha - \xi + \sin \theta - \xi \sin \theta) H + \frac{1}{2}\mu_0 N M_s^2 (2\xi \alpha - \xi + \sin \theta - \xi \sin \theta)^2 + K_u (1 - \xi) \sin^2 \theta.$$  \hfill (14)

2.2.2. Mechanical energy

The total mechanical energy is considered to be composed of elastic strain energy and the energy associated with detwinning. The model parameters required for determination of the mechanical energy of the system are obtained from the stress-strain plot at zero bias field (Figure 2). For characterization of the sensing effect, the material is compressed from its longest length ($\xi = 1$) to beyond the length corresponding to complete stress preferred variant state ($\xi = 0$). As is the case in conventional shape memory alloys (SMAs), the total strain is composed of an elastic and a detwinning component,\textsuperscript{11, 12}

$$\varepsilon = \varepsilon_e + \varepsilon_{tw}.$$  \hfill (15)

The mechanical energy contribution to the thermodynamic potential thus has the form

$$\rho \phi_{mech}(\varepsilon, \xi) = \frac{1}{2} E (\varepsilon - \varepsilon_{tw})^2 + \frac{1}{2} a \varepsilon_{tw}^2,$$  \hfill (16)

in which the detwinning strain is assumed to be linearly proportional to the stress-preferred volume fraction,

$$\varepsilon_{tw} = (1 - \xi) \varepsilon_0.$$  \hfill (17)

Combination of (15) and (17) gives an expression for the elastic strain,

$$\varepsilon_e = \varepsilon - (1 - \xi) \varepsilon_0.$$  \hfill (18)

Parameter $a$ represents a stiffness relating the variation of only detwinning strain to the total stress. It is obtained from the measured model parameters $E$ and $k$, where $E$ represents the Young’s modulus, or the variation of only elastic strain with stress, and $k$ represents the slope of the stress-strain plot in the detwinning region. Thus, $k$ is equivalent to the net stiffness of two springs in series, with the deformations equivalent to the elastic and detwinning strains. Thus, $a$ can be calculated as

$$\frac{1}{a} = \frac{1}{k} - \frac{1}{E}.$$  \hfill (19)

Since $E$ represents an average elastic modulus, it is calculated by taking the average of the initial and final slopes of the stress-strain curve at zero magnetic field. For the material being considered, the measured values of these parameters are $k = 16 \text{ MPa}$ and $E = 1600 \text{ MPa}$.

When the sample is compressed, it is seen that the initial high stiffness region is followed by the low stiffness detwinning region. The stress value at the onset of detwinning at zero magnetic field, $\sigma_{tw}$, is considered to be
a model parameter ($\sigma_{tw}=0.8$ MPa). The detwinning stress is observed to increase with increasing bias field. Combination of relations (16)-(19) yields an expression for the mechanical energy

$$\rho \phi_{mek} (\varepsilon, \xi) = \frac{1}{2} E (\varepsilon - \varepsilon_0 (1 - \xi))^2 + \frac{1}{2} a \varepsilon_0^2 (1 - \xi)^2.$$  \hspace{1cm} (20)

The total free energy is therefore found from (14) and (20) in the following manner,

$$\rho \phi = \rho \phi_{mek} + \rho \phi_{mag},$$  \hspace{1cm} (21)

which in combination with (7) yields a constitutive relation for stress of the form

$$\sigma = E [\varepsilon - \varepsilon_0 (1 - \xi)].$$  \hspace{1cm} (22)

The constitutive relation for magnetization is given by (8).

2.3. Development of Magnetic Parameters

We assume that the processes associated with the rotation of magnetization $\theta$ and evolution of domain fraction $\alpha$ are reversible. This is supported by the fact that the easy- and hard-axis magnetization curves show negligible hysteresis. The easy-axis magnetization process involves evolution of domains which depends on the magnitude of the magnetostatic energy opposing the external magnetic field, whereas the magnetization along the hard axis is due to magnetization rotation. On the basis of reversible processes, the corresponding driving forces lead to zero increase in entropy. Hence, the driving forces themselves must be zero,

$$\pi^\theta = -\frac{\partial (\rho \phi)}{\partial \theta} = 0$$  \hspace{1cm} (23)

$$\pi^\alpha = -\frac{\partial (\rho \phi)}{\partial \alpha} = 0.$$  \hspace{1cm} (24)

Equations (21), (23) and (24) yield a closed form solution for the angle $\theta$, and domain fraction $\alpha$,

$$\theta (H, \xi, \alpha) = \arcsin \left( \frac{2\mu_0 NM^2 \alpha \xi - \mu_0 NM^2 \xi - \mu_0 HM^2}{\mu_0 NM^2 \xi - 2K_u - \mu_0 NM^2} \right)$$  \hspace{1cm} (25)

$$\alpha (H, \xi) = \frac{H}{2NM^2 \xi} + \frac{1}{2}.$$  \hspace{1cm} (26)
Thus, for a given volume fraction $\xi$ and magnetic field $H$, the domain fraction $\alpha$ can be calculated from equation (26). Using this value of $\alpha$, the magnetization rotation angle $\theta$ and magnetization $M$ can be calculated from (25) and (8), respectively. The evolution of volume fraction $\xi$ is discussed in the next section.

For comparison with Hall probe measurements, the magnetic induction is calculated by means of the relation

$$B_m = \mu_0(H + DM),$$

where $D$ is the demagnetization factor in the $x$ direction. A comparison of model results and experimental data for hard axis and easy axis magnetization curves is shown in Figure 3.

### 2.4. Evolution of volume fraction

#### 2.4.1. Loading

During loading of the material, the mechanical energy is given by equation (20). The driving force associated with the evolution of volume fractions is composed of magnetic and mechanical components,

$$\pi^\xi = \pi^\xi_{mag} + \pi^\xi_{mech},$$

with the magnetic and mechanical driving forces given by

$$\pi^\xi_{mag} = \mu_0 HM_s(2\alpha - 1 - \sin \theta) + K_u \sin^2 \theta - \mu_0 N M_s^2(2\xi \alpha - \xi + \sin \theta \sin \theta)(2\alpha - 1 - \sin \theta)$$

$$\pi^\xi_{mech} = -E[\varepsilon - \varepsilon_0(1 - \xi)]\varepsilon_0 + a(1 - \xi)\varepsilon_0^2.$$
From (9) and (23), assuming isothermal process, the Clausius-Duhem inequality is reduced to

\[
\pi \xi \dot{\xi} \geq 0. \quad (31)
\]

During loading, in the initial configuration the sample consists of only one variant preferred by field (\(\xi = 1\)). The loading process takes place with evolution of stress-preferred variants, indicating that \(\xi < 0\). The driving force \(\pi \xi\) should therefore be negative in order for inequality (31) to be satisfied. Thus, the increase of stress-preferred volume fraction \((1 - \xi)\), or decrease in \(\xi\), begins when the driving force reaches the negative value of the critical driving force. The numerical value of \(\xi\) can then be obtained by solving the relation

\[
\pi \xi = -\pi^{cr}. \quad (32)
\]

It is noted that \(\xi\) is restricted so that \(0 \leq \xi \leq 1\). Thus, once \(\xi\) is evaluated at a certain time in the loading process, then the stress \((\sigma)\) and magnetization \((M)\) can be found by equations (22) and (8), respectively.

### 2.4.2. Unloading

The unloading process can be treated independently after the loading model is completed. During unloading, the initial configuration corresponds to the material consisting of one variant preferred by stress \((\xi = 0, 1 - \xi = 1)\). This configuration corresponds to a state where the sample is compressed beyond its natural stress preferred variant length, since the loading is extended even after completion of the detwinning process. Thus, a large negative thermodynamic driving force associated with stress acts on the material along with the driving force associated with the magnetic field, which is always positive. The bias field is always present throughout the unloading process.

Since the detwinning strain is now given by

\[
\varepsilon_{tw} = \varepsilon_0 \xi, \quad (33)
\]

the mechanical energy takes the form

\[
\rho \phi_{mech} = \frac{1}{2} E[\varepsilon - \varepsilon_0 (1 - \xi)]^2 + \frac{1}{2} a\varepsilon_0^2 \xi^2. \quad (34)
\]

The elastic component of the strain is found by subtracting the total detwinning strain incurred during loading, \(\varepsilon_0\), from the original strain value and then accounting for the detwinning strain given by (33). Thus, the elastic component has the form

\[
\varepsilon_e = \varepsilon - \varepsilon_0 + \varepsilon_{tw} = \varepsilon - \varepsilon_0 (1 - \xi). \quad (35)
\]

Employing the procedure detailed in section 2.2.2 for the mechanical energy, the mechanical component of the driving force becomes

\[
\pi^\xi_{mech} = -E[\varepsilon - \varepsilon_0 (1 - \xi)] \varepsilon_0 - a\varepsilon_0^2 \xi. \quad (36)
\]

During unloading, the evolution of fraction \(\xi\) is of interest. Once the detwinning starts, the rate is positive, \(\dot{\xi} > 0\). Thus, the total driving force \(\pi^\xi\) has to be positive in order for the Clausius Duhem inequality to be satisfied. As the total force starts increasing from its lowest negative value, and reaches the positive critical driving force value, the evolution of \(\xi\) is initiated. The value of \(\xi\) can then be found by numerically solving the equation

\[
\pi^\xi = \pi^{cr}. \quad (37)
\]

The constitutive equation for stress, \(\sigma = E\varepsilon_e\), has the same form as (22) due to the elastic strain (35) having the same form as (18).

The procedure of arriving at the values of all parameters with a constant bias field and varying strain \(\varepsilon\) is similar to that outlined earlier. The difference lies in the initial configuration and the terms associated with mechanical energy as stated above. A restriction is placed on the calculated stress that it must be always compressive. Because the sample is not attached to the machine’s cross arm, no tensile stress can be applied to the sample during unloading.
2.5. Sensing model results

The experimental setup used for model parameter identification and validation consists of an electromagnet and a uniaxial stress stage oriented perpendicular to the axis of the magnetic poles. A 6×6×20 mm³ single crystal Ni-Mn-Ga sample (AdaptaMat Ltd.) is placed in the center gap of the electromagnet with its long axis aligned with the stress axis. The sample exhibits a free magnetic-field induced deformation of 5.8 % under a field of 720 kA/m. For the measurements, the material is first converted to a single field-preferred variant by applying a high field, and subsequently compressed at a fixed displacement rate of 25.4 × 10⁻³ mm/sec, and unloaded at the same rate while subjected to different bias fields. The flux density inside the material is measured by a 1×2 mm² transverse Hall probe placed in the gap between one of the magnet poles and the face of the sample. The compressive force is measured by a 200 pounds of force (lbf) load cell, and the displacement is measured by a linear variable differential transducer (LVDT). This process is repeated for several magnetic bias intensities ranging between 0 and 445 kA/m.

The calculated stress-strain plots are compared with experimental measurements in Figure 4. For the various bias fields measured, the model adequately predicts the amount of pseudoelasticity and the strain at which the sample returns to zero stress. As the external bias field is increased, more energy is required to initiate twin boundary motion, resulting in an increase in the detwinning stress with increasing bias field. In prior models by Likhachev and Ullakko and Straka et al., a term known as ‘magnetic field induced stress’ was introduced to account for this effect. Our model requires no adjustments.

The flux density plots shown in Figures 5 and 6 are of interest for sensing applications. The absolute value of flux density decreases with increasing compressive stress. As the sample is compressed from its initial field-preferred variant state, the stress-preferred variants are nucleated at the expense of field-preferred variants. Due to the high magnetocrystalline anisotropy of Ni-Mn-Ga, the nucleation and growth of stress-preferred variants occurs in concert with rotation of magnetization vectors into the longitudinal direction, which causes a reduction of the permeability and flux density in the transverse direction. There is a strong correlation between Figure 2 and Figures 5 and 6 regarding the reversibility of the magnetic and elastic behaviors. Because a change in flux density relative to the initial field-preferred single variant is directly associated with the growth of stress-preferred variants, the flux density value returns to its initial value only if the stress-strain curve exhibits magnetic field induced pseudoelasticity. The model calculations accurately reflect this trend. Further, the model correctly identifies the transition from irreversible quasiplastic to reversible pseudoelastic behavior at a bias field of 368 kA/m. The simulated curves show less hysteresis than the measurements as well as a small non-linearity in the relationship between flux density to strain. The large hysteresis in the measured data is believed to be because of the experimental errors.
3. EXTENSION OF THE MODEL TO ACTUATION EFFECT

Our sensing model can also be used to characterize the actuation behavior of Ni-Mn-Ga. The external field and bias stress constitute the independent variables, with the strain and magnetization being dependent variables. Despite the swapping of variables, the actuator model utilizes the exact same parameters as the sensing model. Further, the actuation model framework is consistent with previous models by Kiefer and Faidley.

The model is formulated by defining the Gibbs energy as thermodynamic potential via Legendre transform,

\[ \rho \phi = \rho \psi - \sigma \varepsilon_e - \mu_0 HM. \]  \hspace{1cm} (38)

This leads to Clausius-Duhem inequality of the form,

\[ -\rho \dot{\phi} + \dot{\sigma}(\varepsilon_e + \varepsilon_{tw}) + \mu_0 H \dot{M} \geq 0. \]  \hspace{1cm} (39)

Following a process similar to that employed to develop the sensing model, we arrive at the constitutive equations

\[ \varepsilon_e = -\frac{\partial \phi}{\partial \sigma}, \]  \hspace{1cm} (40)

\[ M = -\frac{1}{\mu_0} \frac{\partial \phi}{\partial H}. \]  \hspace{1cm} (41)

The Clausius-Duhem inequality reduces to the form,

\[ (-\frac{\partial \phi}{\partial \xi} + \sigma \varepsilon_0) \dot{\xi} \geq 0 \] \hspace{1cm} (42)

\[ \pi^{\xi} \dot{\xi} \geq 0 \] \hspace{1cm} (43)

where the total driving force \( \pi^{\xi} \) can be defined as

\[ \pi^{\xi} = \frac{\partial \phi}{\partial \xi} + \sigma \varepsilon_0 = \pi^{\xi} + \sigma \varepsilon_0 = \pi^{\xi}_{mag} + \pi^{\xi}_{mech} + \sigma \varepsilon_0. \] \hspace{1cm} (44)

The contribution of the magnetic energy to the total Gibbs energy remains exactly the same as that given by (14). The mechanical energy contribution in the Gibbs energy is given by

\[ \phi_{mech} = -\frac{1}{2} S \sigma^2 + \frac{1}{2} a \varepsilon_0 \varepsilon_0. \] \hspace{1cm} (45)

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**Figure 5.** Induction vs. strain model results.

**Figure 6.** Induction vs. stress model results.
The first term represents the elastic Gibbs energy due to stress, while the second term represents the energy due to detwinning. Unlike in the sensing model, the mechanical energy equation remains the same in both loading and unloading.

The initial condition for the actuation process is the sample at its minimum length (\(\xi = 0\)) in the presence of a bias stress \(\sigma\). This bias stress compresses the sample elastically, since the sample is already in the complete stress preferred state. In this situation, the parameters associated with the mechanical energy are the same as those presented for the sensing model. The compliance \(S\) is the inverse of the elastic modulus \(E\), and \(\sigma\) represents the compressive bias stress. When the magnetic field is increased, the driving force due to the field starts acting opposite to the driving force due to stress. The expression for the driving force due to field is the same as (29), and the expression for mechanical driving force is

\[
\pi_{\text{mech}}^\epsilon = -a\varepsilon_0^2\xi + \sigma\varepsilon_0. \tag{46}
\]

When the total driving force exceeds the critical value \(\pi_{\text{cr}}\), twin boundary motion is initiated. The numerical value of volume fraction \(\xi\) can be obtained by solving the relation

\[
\pi = \pi_{\text{cr}}. \tag{47}
\]

The twinning process continues while the applied field is increasing. When the field is decreased, the volume fraction does not decrease with the field immediately, since the total driving force needs to be lower than the negative value of critical driving force. When the applied field becomes sufficiently low, the twin boundary motion in the opposite direction is initiated, and the volume fraction values can be obtained by solving

\[
\pi = -\pi_{\text{cr}}. \tag{48}
\]

The identification of model parameters is conducted by comparison of model results with experimental data published by Murray. The model parameters required for the actuation model are exactly the same as for the sensing model, and for the considered data they are: \(\sigma_{tw}=0.8\) MPa, \(K_u=1.7E5\) J/m\(^3\), \(\varepsilon_0=0.058\), \(E = 850\) MPa, \(k = 14\) MPa, \(M_s = 0.65\) T, \(N = 0.239\). The model results and comparison with experimental data at various bias stresses is shown in Figure 7. The model accurately quantifies the maximum magnetic field-induced deformation at different bias stresses ranging from 0.25 MPa to 2.11 MPa. For most stress values, the model results both for the forward and return path accurately match the measurements. According to the model, the bias stresses of 0.89 MPa and 1.16 MPa can be considered as optimum where the completely reversible behavior is observed with maximum magnitude of strain.

4. CONCLUSION

This paper presents a unified model which can describe the following behavior and interdependence for the complete magnetomechanical characterization of a commercial single crystal Ni-Mn-Ga alloy: (i) sensing: stress and magnetization dependence on strain at constant bias field, and (ii) actuation: strain and magnetization dependence on applied field at constant bias stress. A fixed set of model parameters is used for both cases, which can be readily obtained from two simple magnetomechanical tests. This leads to a simple model with lesser dependence on adjustable parameters and more emphasis on accurate construction of energy terms. The changes needed to shift from one type of model to another are made in the basic thermodynamic framework concerning the use of a given thermodynamic potential, but formulation of specific energy terms remains unchanged.

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Figure 7. Magnetic field-induced strain vs. magnetic field: model results and experimental data at different bias stresses.

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