Morphing structures, defined as body panels that are capable of a drastic autonomous shape transformation, have gained importance in the aerospace, automotive, and soft robotics industries since they address the need to switch between shapes for optimal performance over the range of operation. Laminated composites are attractive for morphing because multiple laminae, each serving a specific function, can be combined to address multiple functional requirements such as shape transformation, structural integrity, safety, aerodynamic performance, and minimal actuation energy. This paper presents a review of laminated composite designs for morphing structures. The trends in morphing composites research are outlined and the literature on laminated composites is categorized based on deformation modes and multifunctional approaches. Materials commonly used in morphing structures are classified based on their properties. Composite designs for various morphing modes such as stretching, flexure, and folding are summarized and their performance is compared. Based on the literature, the laminae in an n-layered composite are classified based on function into three types: constraining, adaptive, and prestressed. A general analytical modeling framework is presented for composites comprising the three types of functional laminae. Model developments for each morphing mode and for actuation using smart material-based active layers are discussed. Results, presented for each deformation mode, indicate that the analytical modeling can not only provide insight into the structure's mechanics but also serve as a guide for geometric design and material selection. [DOI: 10.1115/1.4044269]

Keywords: morphing, laminated composites, multifunctional, analytical model, bistable, curvature, stretching, folding, actuation

1 Background

1.1 Definition. This paper is a review of multifunctional laminated composites that are applicable to morphing structures. The terms that define the scope of this review are explained in Secs. 1.1.1 and 1.1.2.

1.1.1 Morphing Structures. Morphing structures undergo significant deformations in response to actuation relative to their characteristic dimensions in the unactuated state. The large deformations associated with morphing can be in the form of stretching of membranes, flexural deformation in thin plates, or folding at creases. Actuation for morphing is realized through an external force field or through embedded active materials that respond to an external stimulus. Some flora and fauna possess morphing abilities that enable a specific set of needs to be fulfilled in each shape. For example, birds adjust wing morphology to optimize performance in various stages of flight such as take-off, cruising, and landing [1]. Plants such as the Venus fly trap engulf their prey by snapping their leaves together from an open state [2]. Naturally occurring morphing events can be gradual or discontinuous, and are typically self-actuated, which means that the forces are applied from within the structure. These ideas have inspired engineering applications such as morphing aircraft [3,4] and automobiles [5], and soft robotics [6,7].

1.1.2 Multifunctional Laminated Composites. Laminated composites are a construct in which materials, in the form of sheets or plies, are stacked together as layers to achieve material properties that are superior to those of the individual materials. The mechanics of laminated composites is influenced by the dimensions, orientation, and material properties of each of the constituent laminae. The stiffness of passive composites can be tailored to facilitate mechanics such as stretching, flexure, or folding. However, large deformation is associated with tradeoffs in the stiffness to operational loads and actuation work [5]. These tradeoffs can be addressed using smart materials such as piezoelectrics, shape memory alloys (SMA), and active polymers. Smart composites, for example, can be softened to enhance the morphing range with minimal actuation effort and can be stiffened to withstand operational loads with minimal shape deformation. Material systems comprising laminae that are tailored to serve functions such as structural integrity, built-in actuation, intrinsic morphing features (such as bistability), and variable stiffness are defined in this review as multifunctional laminated composites.

1.2 Trends in Morphing Laminated Composites. Interest in the field of morphing structures has grown exponentially in the past two decades (1997–2017). Based on the count returned by the search engine Google Scholar, the number of publications, excluding patents, in the field of composite-based morphing structures are an order of magnitude lower than in morphing structures in general. Also, publications related to morphing laminates or laminated composites are an order of magnitude fewer than morphing composites. The compounded average growth rate for research on morphing structures, composites, and laminates is calculated to be 11%, 14.94%, and 18.59% respectively, highlighting the attractiveness of laminated composites for morphing. The increasing number of patents at a compounded rate of 17.2% in the last two decades indicates growing interest in the adoption of morphing laminates for commercial applications.

1.3 Morphing Applications. Applications that benefit from shape adaptive structures are discussed in this subsection. The emphasis is on aircraft and automobiles where morphing panels enable optimal aerodynamic performance for improved fuel
economy. Other applications include morphing wind turbine blades, soft robotics, and deployable space structures.

1.3.1 Aircraft. Morphing aircraft have received significant attention in recent decades due to the potential for increasing fuel economy and reducing emissions in commercial aircraft, and developing multirole aircraft for defense operations [8]. Bowman et al. [9] numerically evaluated the benefits of morphing the planform shape and profile of the wings of an aircraft. Several wing concepts have been developed with the goal of morphing the span, sweep, wing twist, airfoil camber, and spanwise bending. These developments have been reviewed extensively by Thill et al. [10], Sofla et al. [11], Barbarino et al. [3], and Weisshaar [12]. The ideal morphing configuration for the wings depends on the type of aircraft (commercial, military, cargo, etc.) and its operating requirements such as flight range, payload, speed, and agility.

Examples of laminated structures that have been investigated for aircraft wing morphing include: SMA-reinforced soft composites whose curvature is controlled by actuating the SMAs [13,14] (Fig. 1(a)); corrugated composites [15] with passive and shape memory polymer (SMP) layers [16] that can fix their deformed shape and recover their initial shape (Fig. 1(b)); piezoelectric macrofiber composites (MFC) as laminar actuators for controlling the curvature of a passive structure [17,18]; bistable composites with curvature [19], tailored twist [20], and zero torsional stiffness [21] for morphing between two stable shapes; composite lattice-based cellular structures [22] (Fig. 1(c)), and hybrid laminated composites with high spanwise stiffness and localized low flexural stiffness [23,24] (Fig. 1(d)). Devices based on bistable composites have been developed for air flow control [25,26]. Sun et al. [27] reviewed smart material-based sensors, actuators, structures, and controllers for morphing aircraft.

1.3.2 Automobiles. Aerodynamic drag increases parabolically with an increase in vehicle speed. Drag can be minimized by morphing the vehicle body into the optimal shape at a given speed. A 10% reduction in aerodynamic drag yields a 2% increase in fuel economy [28]. At low speeds, where aerodynamic performance is not critical, morphing panels enhance exterior styling, thermal efficiency, crashworthiness, etc. [29]. Hucho [30] summarized various geometric modifications to the vehicle body that reduce drag. Examples of morphing structures that serve as drag-reducing features include: boat-tail extension [31]; vortex generators [32]; morphing fender skirts [33]; active radiator grills [34]; active underbody cover [35]; deployable side mirrors [30]; deployable air dam [36]; and rear diffuser with a controllable angle of attack [34].

Daynes and Weaver reviewed morphing structure concepts for automobiles up to the year of 2013 [5]. Some of the key morphing vehicle concepts presented since 2013 are: morphing fender on the BMW Next 100 Years concept [37]; boat-tail extension on the Mercedes Benz Intelligent Aerodynamic Automobile concept [38]; inflatable front spoiler on a Porsche 911 Turbo S [39]; and active flaps for engine thermal management on a Ferrari 458 Speciale [40]. Chillara et al. [41] demonstrated a self-supported and self-actuated morphing fender skirt that is dome-shaped at low speed and flattens at high speed (Fig. 2(a)); flattening in curved prestressed composites is realized through laminar SMA wires that contract on heating. SMAs have also been employed to control the curvature of a soft composite-based rear spoiler [42] (Fig. 2(b)). Spiteri et al. [43] developed a bistable origami-folded ram-air intake system for automotive engines. Qamar et al. [44] reviewed materials for human computer interaction that may be applicable to reconfigurable interiors in automobiles. Overall, the number of publications on morphing composites is far fewer for automobiles than compared to aircraft.

1.3.3 Other Applications. The cost of wind energy can be lowered by increasing the generating capacity of wind turbines. Power output can be amplified by increasing rotor diameter but at the cost of higher aerodynamic loads at the blade roots and higher inertial loading. Aereelastic tailoring of rotor blades is an

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Fig. 1 Composite structures in morphing aircraft wings: (a) unmanned aerial vehicle with SMA-actuated soft morphing winglets (reproduced with permission from Han et al. [14]. Copyright 2016 by Elsevier Ltd.), (b) variable camber wing based on a smart composite comprising corrugations filled with SMP (reproduced with permission from Gong et al. [16]. Copyright 2017 by IOP Publishing), (c) twisting wing based on cellular structures laminated to a flexible skin (reproduced with permission from Jenett et al. [22]. Copyright 2017 by Mary Ann Liebert Inc.), and (d) droop-nose demonstrator based on a hybrid composite skin comprising localized soft regions for flexure (reproduced with permission from Rudenko et al. [23]. Copyright 2017 by SAGE Publications).
attractive approach for maximizing power and durability. Lightweight blades with tailored stiffness have low inertial loads and can generate power up to a critical aerodynamic load beyond which they morph into nonfunctional shapes, thereby alleviating stress and improving durability. Lachenal et al. [45] reviewed morphing composite-based structures for wind turbine blades. Commonly studied methods for load alleviation are based on tailoring the bend-twist coupling of a composite blade. They include: multimaterial structures with variable ply thickness [46] and selective stiffeners [47]; and composites with variable stiffness that exhibit multistability [48], zero torsional stiffness [21] (Fig. 3(a)), and active shape change using embedded SMA actuators [49,50].

Soft robots are perceived as robots whose shape and stiffness can be controlled to realize locomotion, safety in human–robot interaction, and dexterous activities like gripping objects and moving them. Composite-based designs of soft robots typically include active materials that are embedded in or laminated to passive layers [51]. Locomotion is achieved by controlling the input to the active material to create flexure or folding in the body [52,53] (Fig. 3(b)). Stiffness control is critical in human–robot interaction where the robot is expected to soften when it comes in contact with a human. Manti et al. [54] reviewed stiffening mechanisms, some of which are relevant in a laminated composites construct. Li and Wang [55] reviewed plant inspired adaptive structures and materials for morphing. Examples of bioinspired composite structures include: a starfish robot based on embedded SMA wire actuators [56]; jellyfish robot based on ionic polymer metal composite actuators [57]; inchworm-inspired robot and turtle-like swimming robot made of soft composite [58,59] (Fig. 3(c)); and fluid-actuated elastomeric robots [60]. Cao et al. [52] presented untethered soft robots that are actuated using soft electrostatic actuators.

Deployable structures are attractive for aerospace applications because they can be packaged in a small volume during launch.
and can be deployed over a large area when in orbit. Examples include solar arrays and boom structures [61]. Fiber-reinforced shape memory polymeric composites are attractive because they can be folded or rolled in one configuration and can be opened or flattened when heated. These materials have been developed under the name of elastic memory composites [62]. Wang et al. [63] developed truss-like structures based on composites with soft elastomeric hinges that are actuated by shape memory alloy wires. Bistable composite tape-springs can be deployed from a rolled-up bulk state into a boom structure [64,65].

1.4 Classification of Morphing Laminated Composites.

Laminated composites developed for morphing structures are categorized based on a keyword search as shown in Fig. 4. A comparison of deformation modes of the laminates shows that flexure, including twisting and coiling, has been studied the most (Fig. 4(a)). Interest in special features such as multistability and embedded actuation has contributed to this trend. Figure 4(b) shows that more than half of the work on morphing laminates includes the notion of actuation. The rest of the literature is on identifying new material combinations for morphing and tailoring the anisotropy of passive composites. Our literature search on variable stiffness included passive composites with functionally graded stiffness and active composites that can change their stiffness on command. Results show that there has been relatively little work on variable stiffness structures compared to actuation. In Sec. 2, various active and passive materials that have been used in the design of morphing laminated composites are summarized and the rationale behind their selection is discussed.

2 Materials for Morphing Composites

The choice of materials for a given morphing mode depends on material modulus, strain capability, and anisotropy. The upper limits of Young’s modulus and usable strain of various passive and active materials commonly used in morphing composites are shown in Fig. 5. Soft materials with high strain capability are candidates for stretchable membranes or skins. Kikuta [66] compared various stretchable material candidates including elastomers such as silicone rubber and polyurethanes, copolyesters, and woven fabrics such as Spandex. These materials are capable of strain on the order of 100%. Among the materials evaluated, silicone rubber has the highest strain capability, minimal residual strain on unloading, and can be used as a matrix material in reinforced composites.

Anisotropy in stretchable materials can be tailored to achieve a target magnitude and direction of strain and stiffness. For example, stretching in an elastomer can be confined to one direction by reinforcing the material with unidirectional fibers such as carbon or aramid in a direction orthogonal to the stretched direction. Such fiber-reinforced elastomers, also known as elastomeric matrix composites (EMC), exhibit near-zero in-plane Poisson’s ratio due to the restriction offered by the fibers [67,68]. The out-of-plane stiffness of EMC membranes can be augmented by applying prestress in the fiber-direction; it is often a structural requirement that the out-of-plane stiffness of stretchable skins be high. Besides enabling changes in surface area, stretchable skins provide restoring forces that aid flexure and folding when installed in a prestressed configuration [69,70].

Ductile materials that have high modulus and low strain relative to elastomers can be used as laminae that provide structural integrity to the composite. These laminate have high in-plane stiffness to restrict stretching but low out-of-plane stiffness to enable flexure. For example, thin sheets of isotropic materials such as steel, aluminum, and nylon can be paired with an active layer that shrinks or expands to create a smooth curvature in the composite [41,70,71]. Brittle materials such as epoxy-based polymers can be reinforced with fibers to create thin composites for morphing. When cured at elevated temperatures, these fiber-reinforced polymeric composites (FRP) can exhibit multiple stable curved shapes at room temperature, depending on the composite’s geometry, fiber-orientation, and curing temperature [72–74].

Fig. 4 Categorization of 2300 publications on morphing laminated composites from 1997 to 2017 based on a Google Scholar search. Trends related to: (a) deformation modes and (b) multifunctional designs.

Fig. 5 Materials commonly used in morphing laminated composites.
Materials whose stress-states can be controlled using external inputs such as heat, electric field, and light are termed smart materials. When paired with passive layers, smart laminae can create a strain mismatch that leads to flexure or folding. Piezoceramics can generate up to 0.2% strain in response to an applied voltage [75]. These ceramics are commercially available in the form of piezoelectric MFC. For large deflections, SMA provide sufficient strain capability (up to 8%) to serve as embedded actuators. SMAs transform from their detwinned Martensite phase to the Austenite phase when heated beyond their transformation temperature at a given stress state. SMPS exist in their rubbery phase to the Austenite phase when heated beyond their transformation temperature (T_g). SMPs can undergo a strain of up to 100% (rubbery) [77,89–91]. Smart laminae can also be realized through fluid channels created in compliant materials. Stretching and flexure can be achieved by modulating fluid pressure [76], whereas stiffness can be controlled by trapping the fluid in the adaptive layer [77,78].

<table>
<thead>
<tr>
<th>Feature</th>
<th>Actuation source</th>
<th>Strain capability</th>
<th>Actuation energy density (kJ/m³)</th>
<th>Flexural stiffness (MPa/m)</th>
<th>Advantages (■) and limitations (□)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMCS [67,79,80]</td>
<td>External force</td>
<td>&gt; 100%</td>
<td>1.25</td>
<td>0.147</td>
<td>■ Zero Poisson’s ratio for 1D morphing</td>
</tr>
<tr>
<td>Elastomer bonded to cellular support layer [68]</td>
<td>External force</td>
<td>&gt; 100%</td>
<td>0.535</td>
<td>4.8</td>
<td>■ Tailorable fiber distribution</td>
</tr>
<tr>
<td>Honeycombs sandwiched between stretchable laminae [81–84]</td>
<td>External force</td>
<td>Up to 12 times material limit</td>
<td>1130</td>
<td>16.3</td>
<td>■ Low out-of-plane stiffness</td>
</tr>
<tr>
<td>Compliant laminae with sandwiched corrugations [85–88]</td>
<td>External force</td>
<td>10%</td>
<td>52</td>
<td>0.289</td>
<td>■ Higher out-of-plane stiffness than pure skin</td>
</tr>
<tr>
<td>Made of semicircular Corrugations [87]</td>
<td>2 × feature radius</td>
<td>N/A (along feature)</td>
<td>15,400</td>
<td>3.9</td>
<td>■ Higher actuation energy than pure skin</td>
</tr>
<tr>
<td>Fluid channels embedded in compliant media [77,89–91]</td>
<td>Pressurized fluid</td>
<td>Up to 20%</td>
<td>2.5 (with trapped fluid), 0.1 (variable fluid volume)</td>
<td>50</td>
<td>■ High out-of-plane stiffness</td>
</tr>
<tr>
<td>Fiber reinforcement in SMPs [92–95]</td>
<td>Resistive heating</td>
<td>5% (glassy)</td>
<td>1160</td>
<td>1.4</td>
<td>■ Potential for nonuniform stretching through functionally graded cells</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt;100% (rubbery)</td>
<td>19.5</td>
<td>0.043</td>
<td>■ Penal on composite thickness</td>
</tr>
</tbody>
</table>

3.1 Stretching. Stretchable composites are material systems that provide a change in surface area. The requirement for stretchable morphing systems is to have low in-plane stiffness but high out-of-plane stiffness. Various designs of stretchable composites are summarized in Table 1. Fiber-reinforcement of elastomers is a versatile approach for the tailoring of stretchable features in membranes. Murray et al. [67] developed zero-Poisson’s ratio elastomers by reinforcing the matrix with unidirectional fibers in the direction orthogonal to the morphing direction. Kubert et al. [68] improved the out-of-plane stiffness of these membranes by reinforcing them with a zero Poisson’s ratio honeycomb (Fig. 6(a)). Geometric features such as corrugations enable low in-plane stiffness in one direction and high bending stiffness in one or two directions [86] (Fig. 6(c)). Active elements such as pneumatic muscle fibers have been installed in a stretchable skin to control the strain of the composite [90] (Fig. 6(d)). When used in conjunction with valves, these muscles can also provide stiffness control [77]. Smart materials such as shape memory polymers are stiff in their glassy state and stretchable up to 400% in their rubbery state [96]. The strain in these polymers can be fixed by cooling them below their glass transition temperature in the stretched shape and

can be recovered by heating them beyond the transition temperature.

3.2 Flexure. Flexible smart composites can be realized by combining a compliant active layer with controllable strain to a flexible but inextensible layer. Designs for flexural laminates are summarized in Table 2. Corrugations can be used to create composites with bilinear flexural stiffness that is characterized by the contact between corrugated unit cells [97]. Prior to contact between the unit cells, flexural stiffness is a function of cell geometry, whereas after contact, it is a function of the material properties of the composite. Cellular structures with tailored Poisson’s ratio have been developed to create flexure configurations such as synclastic, anticlastic, and cylindrical curvatures. An example of a cylindrical composite based on shape memory polymers reinforced with zero-Poisson’s ratio cellular structures is shown in Fig. 7(a). Smart flexible skins can be created by installing an active material at an offset from the neutral axis; an example of a flexible skin with SMAs fused to the matrix is shown in Fig. 7(b).

Initially curved composites can be created by incorporating residual stress through thermal [72] or mechanical [70] means; Daynes and Weaver reviewed methods for applying prestress to composite structures [127]. The curved structure can be flattened using active laminae such as fluid channels as shown in Fig. 7(c); when actuation is turned off, the composite returns to its curved shape due to the intrinsic restoring stress.

The most commonly studied bistable composites are FRP laminates that are cured at an elevated temperature [128]. Residual stress develops due to the mismatch in thermal contraction between the matrix and the fibers. In laminates with an unsymmetric fiber lay-up, the resulting cylindrical curvatures have nonparallel axes. Daynes et al. [117] developed bistable laminates with symmetric curved shapes by applying prestress to the fibers during the curing process (Fig. 7(d)). Methods for amplifying the magnitudes of the bistable shapes include the sandwiching of metallic laminae with FRP laminae [71,115]. Chilla and Dapino [124,125] developed bistable laminates that are mechanically prestressed by bonding stretched fiber-reinforced elastomeric laminae on opposite faces of an inextensible lamina (Fig. 7(e)). The stable curved shapes are weakly coupled when the prestressed laminae are orthogonal to each other. A key difference between the stress-bias and thermal-curing mechanisms is that mechanical prestress can be applied locally whereas thermal stress is applied globally. Localization of prestress enables the inclusion of multifunctional laminae in the composite. Methods for the actuation of bistable composites include the use of smart laminae such as piezoelectric MFC [129], shape memory alloys [118,126], and thermal loading [119].

3.3 Folding. Foldable thick composites can be created by incorporating low flexural stiffness in a localized region that defines a crease. The modulus and/or thickness of the crease is much lower than that of its faces. Lang et al. [130] reviewed various techniques for thickness accommodation in origami-folded structures. Various mechanisms for foldable composites are summarized in Table 3. Folded composites can be categorized as: structures with zero-width creases and structures with finite-width creases. Zero-width creases yield sharp folds that can be created using paper or mold-cured materials whereas finite-width creases yield folds that have high localized curvature. Boncheva and Whitesides [132] developed sealed elastomer-coated origami paper structures that can be deployed through pneumatic inflation (Fig. 8(a)). Sharp creases can be obtained by curing laminae using materials such as silicone rubber in molds made in the folded shape. Daynes et al. [134] demonstrated multistable origami shapes using elastomeric laminae that have acrylonitrile butadiene...
Folds in a laminate with constraining and active layers can be actuated using mechanisms similar to those described in flexible multilayer composites [111–114]. Premade origami sheets can be laminated with a sandwiched fluidic layer to create origami structures that have controllable shape and stiffness [78].

**4 Modeling Methods**

The theory underlying various morphing structure concepts is summarized in this section. In Sec. 4.1, we present a generic analytical modeling framework for thin plates (2D structures) based on classical laminate theory. Conditions of plane-stress and strain are assumed and the variations in thickness and through-thickness
Fig. 7 (a) SMPs reinforced with zero-Poisson’s ratio honeycombs (reproduced with permission from Huang et al. [110]. Copyright 2017 by Elsevier Ltd.), (b) elastomer reinforced with SMA wires that are fused to the matrix (reproduced with permission from Feng et al. [107]. Copyright 2015 by Elsevier Ltd.), (c) curved composites with intrinsic pressure actuation and prestress [70], (d) bistable buckled laminates with symmetric curved shapes (reproduced with permission from Daynes et al. [117]. Copyright 2008 by Elsevier Ltd.), and (e) mechanically prestressed bistable laminates with weakly coupled curved shapes [124].

Table 3 Summary of foldable laminated composites

<table>
<thead>
<tr>
<th>Feature</th>
<th>Actuator/active element</th>
<th>Analyzed feature</th>
<th>Actuation energy densitya (kJ/m³)</th>
<th>Crease stiffnessa (N mm/rad)</th>
<th>Advantages (■) and limitations (□)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lamina emergent torsional joints [131]</td>
<td>External force</td>
<td>Crease</td>
<td>N/A</td>
<td>14,395</td>
<td>Kinematic locking for high stiffness</td>
</tr>
<tr>
<td>Mechanically prestressed folds [132,133]</td>
<td>External force</td>
<td>Crease</td>
<td>1.875</td>
<td>N/A</td>
<td>Flexibility for folding</td>
</tr>
<tr>
<td>Multistable origami composites [134–136]</td>
<td>External force, pressurized fluid</td>
<td>Folded structure</td>
<td>18</td>
<td>19,100</td>
<td>Rapid switching between folded shapes</td>
</tr>
<tr>
<td>Fluid layer sandwiched between origami sheets [78]</td>
<td>Pressurized fluid</td>
<td>1D tensile origami structure</td>
<td>5300 (open chamber), 18,580 (trapped fluid)</td>
<td>0.91 (open), 3.19 N/mm (trapped)</td>
<td>Not realizable when fold faces are rigid</td>
</tr>
<tr>
<td>Pressurized elastomeric folded structures [137]</td>
<td>Pressurized air</td>
<td>1D contracting origami structure</td>
<td>375</td>
<td>N/A</td>
<td>Compatible with nastic actuation</td>
</tr>
<tr>
<td>SMP-activated origami folds [138–140]</td>
<td>Resistive heating of SMPs</td>
<td>Crease</td>
<td>N/A</td>
<td>8.2</td>
<td>Shapes can be fixed and recovered</td>
</tr>
<tr>
<td>SMA activated origami folds [141–143]</td>
<td>Torsional SMA wire actuator</td>
<td>Hinge with an embedded SMA</td>
<td>3750</td>
<td>0.143</td>
<td>Attractive for localized actuation</td>
</tr>
<tr>
<td>Electro- and magneto-active folds [144–146]</td>
<td>Electric or magnetic field</td>
<td>Four-layer terpolymer crease</td>
<td>8.7</td>
<td>8.1</td>
<td>Composites respond to multiple fields</td>
</tr>
</tbody>
</table>

*aCalculated using the reported values of folding torque, folding range, or actuation energy.
shear strain are considered to be minimal. The model is meant for thin to moderately thick composites with geometric nonlinearities. Strains are described using nonlinear expressions based on the Lagrangian formulation and von Karman’s hypothesis is used to account for moderate rotations. The deformed shapes are calculated as a function of actuation and external forces using strain energy minimization. Under suitable kinematic assumptions (see Table 4), the model is applicable to stretchable, flexible, or foldable composites. The developments in the modeling of composites for various morphing modes is presented in Sec. 4.2.

### 4.1 General Analytical Laminated-Plate Model

#### 4.1.1 Laminate Strain Formulation

Reddy [152] showed that strains of an arbitrary point \((x, y)\) on the composite are written in accordance with von Karman’s hypothesis as

\[
\begin{align*}
\frac{\partial v}{\partial x} &= 0, \quad \frac{\partial v}{\partial y} = 0, \\
\frac{\partial u}{\partial x} &= 0, \quad \frac{\partial u}{\partial y} = 0,
\end{align*}
\]

with

\[
\begin{align*}
u(x, y, z) &= u_0(x), \\
v(x, y, z) &= v_0(y),
\end{align*}
\]


Table 4 Strain models for various morphing mechanics

<table>
<thead>
<tr>
<th>Mechanics</th>
<th>Example cases</th>
<th>Assumptions</th>
<th>Displacements</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stretching</td>
<td>Anisotropic skins</td>
<td>(\frac{\partial v}{\partial x} = 0, \frac{\partial v}{\partial y} = 0)</td>
<td>(u(x, y, z) = u_0(x), v(x, y, z) = v_0(y))</td>
<td>[148]</td>
</tr>
<tr>
<td></td>
<td>Zero Poisson’s ratio skins</td>
<td>(\frac{\partial v}{\partial x} = 0, \frac{\partial v}{\partial y} = 0, \frac{\partial u}{\partial x} = 0, \frac{\partial u}{\partial y} = 0)</td>
<td>(u(x, y, z) = u_0(x), v(x, y, z) = v_0(y))</td>
<td>[67]</td>
</tr>
<tr>
<td>Flexure</td>
<td>Curved composites, bistable curvatures</td>
<td>(\frac{\partial u}{\partial x} = 0, \frac{\partial v}{\partial y} = 0, \frac{\partial u}{\partial y} = 0, \frac{\partial v}{\partial x} = 0)</td>
<td>(u(x, y, z) = u_0(x) - \frac{\partial w_0}{\partial x}, v(x, y, z) = v_0(y) - \frac{\partial w_0}{\partial y})</td>
<td>[73, 149, 74, 150]</td>
</tr>
<tr>
<td>Folding</td>
<td>Smooth, curved folds</td>
<td>(\frac{\partial u}{\partial x} = 0, \frac{\partial v}{\partial y} = 0, \frac{\partial u}{\partial y} = 0, \frac{\partial v}{\partial x} = 0)</td>
<td>(u(x, y, z) = u_0(x) - \frac{\partial w_0}{\partial x}, v(x, y, z) = v_0(y) - \frac{\partial w_0}{\partial y})</td>
<td>[151, 133]</td>
</tr>
</tbody>
</table>

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JANUARY 2020, Vol. 72 / 010801-9

Fig. 8 (a) Inflatable paper-elastomer origami composites (reproduced with permission from [137]), Copyright 2017 by Wiley), (b) elastomer-acrylonitrile butadiene styrene composites with multistable origami shapes (reproduced with permission from Ref. [134]), Copyright 2017 by IOP Publishing), (c) folds created through selective activation of a SMP layer (reproduced with permission from Ref. [140]), Copyright 2017 by IOP Publishing), and (d) stress-biased composites with folded stable shapes [133].

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The simplified strain expressions for each morphing mode are obtained using the assumptions listed in Table 4. Displacements $u$, $v$, and $w$ of an arbitrary point in the composite are related to the geometric midplane’s displacements $u_0$, $v_0$, and $w_0$ in the $X$, $Y$, and $Z$ directions, respectively, as shown in Table 4. Strains of an arbitrary plane $z$ are obtained by substituting the displacements $u$, $v$, $w$ into Eqs. (1)–(3), leading to the relations

$$\epsilon_x = \epsilon_0^x + z\kappa_x,$$

$$\gamma_{xy} = \gamma_{xy}^0 + 2z\kappa_{xy},$$

$$\epsilon_y = \epsilon_0^y + z\kappa_y$$

(4)

where $\epsilon_0^x$ and $\epsilon_0^y$ are the in-plane axial strains, $\gamma_{xy}^0$ is the in-plane shear strain, and $\kappa_x$, $\kappa_y$, and $\kappa_{xy}$ are the curvatures and twist, respectively, of the geometric midplane. The $\kappa$ terms would be zero in the case of stretching.

4.1.2 Strain Energy Computation. The elastic potential energy components can be simplified based on the assumptions of plane stress and strain. The strain energy $(U_T)$ of the composite can be expressed as a function of the geometric and material properties of the laminate as

$$U_T = \int \left[ \frac{1}{2} Q_{11}\epsilon_x^2 + Q_{12}\epsilon_x\epsilon_y + \frac{1}{2} Q_{22}\epsilon_y^2 + Q_{16}\gamma_{xy}\epsilon_x + Q_{26}\gamma_{xy}\epsilon_y + \frac{1}{2} Q_{66}\gamma_{xy}^2 \right] dV,$$

(5)

where $\{Q_{ij}\}$ are the plane-stress-reduced stiffness parameters [152].

Residual stress generated through thermal or mechanical sources influences a composite’s strain energy. Therefore, the energy term $U_T$ should reflect the contribution of these sources. Thermal strain can be modeled by substituting the strains $\epsilon_0^x$, $\epsilon_0^y$, and $\gamma_{xy}$ with $\epsilon_0 - \Delta T \epsilon_0$, $\epsilon_0 - \Delta T \epsilon_0$, and $\gamma_{xy} - \kappa_{xy} \Delta T$, respectively; $\kappa_x$, $\kappa_y$, and $\kappa_{xy}$ are the coefficients of thermal expansion and $\Delta T$ is the difference between the curing and operating temperatures. The $\Delta T$ terms are ignored in the calculation. Modeling of thermal strains is typical in the calculation of the stable shapes of thermally cured bistable FRP composites [73].

Residual stress created using mechanically prestressed laminae can be modeled using strain components (e.g., $\epsilon_x$) that are expressed as the difference between the prestrain ($\epsilon_0x$) applied prior to lamination and the composite’s strain (e.g., $\epsilon_0 - \epsilon_0x$). This approximation is valid only if the applied prestrain is within the limits of the layer’s linear material response. In highly stretchable laminae such as elastomers, the material response is nonlinear and the experimentally measured strain corresponds to a nonlinear expression. Material nonlinearity can be included by expressing the strain energy as the area under the stress–strain curve with $\epsilon_0 - \epsilon_0x$ as the independent variable.

4.1.3 Work Done on the Composite. The work done by forces and moments generated by actuators and operational loads can be calculated in the variational form. Actuation forces have been modeled in various configurations such as transverse, axial, and embedded, to study the composite deformations [126,153]. In composites where the actuators are integral to the structure, such as fluid-filled chambers, variational work can be expressed directly as a function of composite strains and fluid pressure. In case of a rectangular fluid channel, volumetric change can be expressed as $(1 + \epsilon_y)(1 + \epsilon_x) - 1$, assuming constant thickness under inflation [70].

The equilibrium shapes of the laminate are obtained by minimizing the net energy using a variational approach. To this end, the strains and out-of-plane displacement are calculated simultaneously solving a set of $n$ nonlinear equations, corresponding to $n$ unknown displacement coefficients. In the absence of any explicit boundary conditions, the Newton–Raphson approach can be employed to numerically approximate the equilibrium shapes of the laminate. To obtain solutions corresponding to the stable shapes, a constraint on the Jacobian of the system of equations is included in the model. The Jacobian matrix is computed with respect to $\epsilon_y$ and is required to be positive definite in order to have a stable solution. In cases where elastic or rigid boundary conditions must be satisfied, a constrained optimization solver such as $fmincon$ in MATLAB can be used to solve the equations.

4.2 Modeling of Specific Deformation Modes

4.2.1 Stretchable Composites. The in-plane response of highly stretchable composites, such as EMC, was modeled by Peel and Jensen [148]. The strain of an EMC measured in a uniaxial tensile test (Fig. 9(a)) corresponds to a nonlinear strain expression.

Fig. 9 (a) Measured stress–strain curve of an EMC with zero in-plane Poisson’s ratio. (b) Analytical modeling of the out-of-plane stiffness of a flexible matrix composite (reproduced with permission from Murray et al. [67]. Copyright 2010 by SAGE Publications).
of the form $e_x = (\partial u/\partial x) + 0.5(\partial u/\partial x)^2$. For compatibility with classical laminate theory, the nonlinear strain expression was written in linear form. Stress is calculated incrementally at discrete values of linear strain using an experimentally obtained pointwise modulus and described in terms of Ogden coefficients [154].

Modeling of the out-of-plane deformation of stretchable composites is critical in developing skins with high transverse stiffness. Murray et al. [67] used a trigonometric shape function of the form $A \sin(\pi x/a)\cos(\pi y/b)$ to model the deflection of a rectangular membrane constrained at edges $x = \{-a, a\}$ and $y = \{-b, b\}$ (Fig. 9(b)). Murugun et al. [80] developed finite element based multi-objective optimization models to calculate optimal fiber distributions such that the composites require minimal actuation force and have maximum out-of-plane stiffness. Advanced optimization models have been developed based on genetic algorithms to include system parameters such as the number of ribs supporting a stretchable membrane [79].

Laminae with geometric features such as cells and corrugations have been developed as structural reinforcements for stretchable skins. Olymipo et al. [81,84] showed that cellular cores have superior strain and stiffness capabilities compared to their constituent materials. In-plane deformation of cellular structures was modeled based on the cellular material theory developed by Gibson and Ashby [155]; the theory describes cell mechanics in terms of small axial and shear deformations of a beam element in a unit cell. Transverse shear deformations were included in the modeling of flexure in these porous cellular cores [81,83]. Cellular structures are strain-limited since their in-plane stiffness increases with increase in in-plane strain. Bai et al. [87] modeled stretchable laminated corrugated panels as thin inextensible curved beams; stiffness of these panels increases as the corrugations flatten and reaches a maximum in the flat state. For corrugations made using FRP composites, the stiffness matrix obtained from laminate theory has been combined with energy expressions to calculate deformations using Castigliano’s theorem [88].

The modeling of adaptive stretchable composites is specific to the type and configuration of the active or smart material. As an example, Philen’s group [77,156] modeled fluidic actuators for stretching as two cylindrical laminae where the inner elastomeric lamina is isotropic and the outer fiber-reinforced lamina is homogeneous and orthotropic. Smart material-based stretchable-composite modeling includes the homogenized constitutive model for an SMA-reinforced SMP by Song et al. [95].

4.2.2 Flexible Composites. The shapes of flexible composite plates with constant curvature can be calculated by considering the in-plane displacements $u_0(x, y)$ and $v_0(x, y)$ of the geometric midplane to be cubic polynomials and the out-of-plane deflection $w_0(x, y)$ to be a quadratic polynomial containing terms with even power [73,74]. Modeling of bistable composites has received special attention given the design possibilities based on the nonlinear snap-through phenomenon. Beginning with Hyer’s analytical model [73], there have been several studies by their group on bistability in thermally cured asymmetric FRP laminates through analytical modeling [74,149,157] and finite element methods [158].

Established on the energy-minimization-based analytical modeling, there have been several new designs and configurations of bistable composites [115,159–161] including: symmetric laminates [116], buckled laminates [117], sandwiched metallic cores [71]; and initially curved thermally loaded composites [119]. Chiliera and Dapino’s model [124] of mechanically prestressed bistable composites includes material and geometric nonlinearities associated with large strain in the prestressed elastomeric layers.

Constant curvature is often assumed in the design of actuation systems for curved composites. Bistable composites actuated by piezoelectric MFC are modeled by including the MFC’s stiffness in the potential energy term and voltage input in the form of work done on the composite [123,162]. SMAs have been modeled as embedded actuators because their strain capability of 6% is in the same range as the in-plane strains produced in curved composites. Commonly modeled as wires, these actuators contract upon heating, thereby creating flexure. The SMA returns to its original length when deactivated due to the internal stress in the bent composite. Dano and Hyer [153] modeled SMAs in a tendon configuration. Composite shape was calculated as a function of the forces applied by the wire at its ends. The forces are input as stress, into the SMA’s constitutive equation (Brinson model [163]) to calculate actuation temperature. Analytical energy-minimization models and finite element models have been presented for SMA-actuated bistable composites [105,118].

Quadratic approximations of displacement polynomials and the assumption of constant-curvature are accurate for the calculation of stable shapes but not for modeling transitional phenomena such as snap-through loads [164] and the limits of bistability [165,166]. Cantera et al. [167] relaxed the assumptions by considering non-uniform curvature and a uniform through-thickness normal strain. Through a model-order study, Pirrera et al. [168] showed that increasing the polynomial order of the displacement functions results in improved accuracy in the calculation of bifurcation points (Fig. 10(a)) and snap-through loads. Beyond seventh-order, the improvement in accuracy is marginal when compared to the increase in computational cost. For asymmetric rectangular laminates with cylindrical stable shapes, the following assumptions were made to simplify the complete displacement polynomials: the composite is clamped at the center; $u_0$ is odd in $x$ and even in $y$; $v_0$ is even in $x$ and odd in $y$; and $w_0$ is even in $x$ and $y$ and is zero at the center. The displacements are nondimensionalized to eliminate inaccuracies related to numerical conditioning. Using

Fig. 10 (a) Model-order study of shape bifurcation in thermally cured bistable FRP laminates (reproduced with permission from Pirrera et al. [168]. Copyright 2010 by Elsevier Ltd.), (b) actuation work ($W$) requirements for the snap-through of bistable composites or the flattening of curved prestressed composites; calculations are performed for mechanically prestressed laminates with dimensions 152.4 × 152.4 × 0.127 mm (see Ref. [125] for details).
seventh-order polynomials, a study of actuation-force cases shows that the energy required for embedded actuation is an order of magnitude higher than in the case of an external axial force (Fig. 10(b)). However, smart materials can deliver sufficient force and stroke to make embedded actuation a practical approach for curved composites.

4.2.3 Foldable Composites. Folds in laminated composites have been modeled under the assumption of zero in-plane strain and constant global curvature at the crease [151]. Individual creases are modeled under the assumption that the faces are rigid and flat. Fold angle is then calculated using strain energy minimization as a function of material, geometric, and actuation parameters. The assumption of zero in-plane strain is valid only for relatively stiff materials and thin creases. For generic creased composites, however, the modeling should reflect the sharp localized deformations described by folds. Mattioni et al. [150] developed a piecewise-continuous displacement model to calculate localized deformations in composites in the presence of elastic boundary conditions. Using a similar approach, Chillara and Dapino [133] modeled creased composites that have a folded shape due to mechanical prestrain applied to specific laminae in a nonparallel orientation across a crease. Figure 11 shows the relation between fold angle and crease width for the case with prestrain orthogonal to the crease line. To include the effects of the elastic boundary conditions formed by the adjoining faces, the crease is assumed to have nonuniform curvature described by a biquadratic out-of-plane displacement polynomial. While the assumption of constant crease curvature is sufficient to model a fold due to orthogonal prestrain, higher-order polynomials are required to model folds due to nonorthogonal prestrain.

Micromechanics studies of folded reinforced composites have shown that fibers break under large flexure in a stiff matrix such as epoxy [169]. In a soft matrix, however, the fibers buckle without breaking due to the low shear stress [170,171]; the fiber layers that are under compressive stress undergo microbuckling. Francis et al. [172] reviewed analytical models for the microbuckling of the fibers in the soft matrix composites and presented an expression for the wavelength of the buckled fibers. López Jimenez and Pellegrino [171] presented a finite element model of folded fibers in a hyperelastic matrix. Results showed that the fibers undergo in-plane buckling, out-of-plane deformations, and exhibit uniform curvature in the folded region.

The development of origami-folded composites is typically based on experiments and FE analysis of a unit cell of a periodic structure. Parametric studies have been presented to guide the design of various functions such as bistability [136], intrinsic actuation [78,137–139], and 2D–3D transitions through lamina-emergent mechanisms [131].

5 Summary and Outlook
This review highlights the emergence of laminated composites as an attractive design platform for morphing structures. For structural applications, the properties of polymer-based laminated composites have been enhanced through fiber-reinforcements and inclusions in various configurations to provide high stiffness and high strength-to-weight ratio. In the past few decades, this design approach has been employed toward the development of composites that can undergo large deformations and thereby serve as morphing elements. By tailoring the anisotropic material properties of the composite, various morphing modes such as stretching, flexure, twisting, coiling, and folding can be achieved. Further, smart or active layers can be integrated into composite structures to achieve shape and stiffness control (actuation). Emerging applications for these structures in the aerospace, automotive, and robotics industries provide a wide range of opportunities for innovation in design, scalability, lightweighting, safety, and performance. The trade-off with morphing composites is that they should be sufficiently stiff to withstand operational loads, but should provide maximal deformation while consuming minimal actuation energy.

Several new approaches have been presented for tailoring the stiffness of passive materials to achieve large deformations. Smart materials such as piezoelectrics and shape memory alloys and active features such as fluid-filled cavities have been commonly used as shape control elements because of the potential for embedding them or including them as laminae. In this review, a general design framework is presented for composite structures based on three types of laminae, viz., constraining, adaptive, and prestressed (stress-biased). Based on the contributions of several researchers, an analytical modeling framework is also presented as a basis for the design of morphing laminated composites. Given the variety in the available designs, it is challenging to perfectly generalize the modeling approach since the finer modeling details are a function of the choice of laminae (cellular, corrugated, etc.) and laminate configuration.

Analytical modeling of morphing laminates has been predominantly in the direct form where the deformations and shapes are calculated from a known set of material and geometric properties. However, there is a need for analytical tools based on an inverse approach that guide the selection and design of materials and laminate structure based on a set of known deformed shapes. This type of modeling would entail the use of optimization tools, neural networks, etc., under a given set of material constraints. A deeper understanding of composite scalability is also needed because there are several opportunities for the design of nonlinear characteristics based on material and geometric nonlinearities. For example, the nonlinear response of bistable or buckled composites...
is a function of their shape and size and can be tailored to mini-
mize the energy required to actuate the structure. Modeling of
adaptive laminae often involves multiple physics such as struc-
tural mechanics, fluid-structure interaction, and heat transfer.
In this scenario, finite element analysis is more efficient relative
to analytical modeling. This review does not discuss finite element
analysis efforts in detail due to the great variety in designs and
design-specific models in the literature.

Despite a wide range of designs on multifunctional composites,
the field of morphing structures is still emerging. In the field of
morphing aircraft, there have been significant advancements in
morphing unmanned aerial vehicles and a few technologies have
matured into applications on commercial aircraft. In the automo-
tive industry, recent concept-vehicles have featured morphing
panels, but the research is in the early stages. Compliant-robotics
applications have evolved rapidly and there are promising con-
cepts based on multifunctional composites. The challenges com-
mon to all fields are design scalability, repeatability of morphing
response, durability (e.g., viscoelastic effects such as creep in
compliant materials), and tolerance to extreme conditions. In this
scenario, multifunctional laminated composites are promising
because of the possibility for creating hybrid structures with spe-
cial geometric and material properties to address the various chal-
enges that lay in the path of realizing morphing structures in
real-world applications.

Acknowledgment

Financial support was provided by the member organizations of
the Smart Vehicle Concepts Center,2 a Phase III National Science
Foundation Industry-University Cooperative Research Center
under grant NSF IIP 1738723.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>CFRP</td>
<td>carbon fiber-reinforced polymer</td>
</tr>
<tr>
<td>EMC</td>
<td>elastomeric matrix composite</td>
</tr>
<tr>
<td>FE</td>
<td>finite element</td>
</tr>
<tr>
<td>FRP</td>
<td>fiber-reinforced polymer</td>
</tr>
<tr>
<td>MFC</td>
<td>macrofiber composite</td>
</tr>
<tr>
<td>$q_0$</td>
<td>reduced transformed stiffness coefficients</td>
</tr>
<tr>
<td>SMA</td>
<td>shape memory alloy</td>
</tr>
<tr>
<td>SMP</td>
<td>shape memory polymer</td>
</tr>
<tr>
<td>$T_g$</td>
<td>glass transition temperature</td>
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<tr>
<td>$U_f$</td>
<td>strain energy of the composite</td>
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<tr>
<td>$x$, $y$, $z$</td>
<td>coordinates</td>
</tr>
<tr>
<td>$X$, $Y$, $Z$</td>
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<td>$\alpha$, $\beta$, $\gamma$</td>
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<td>$\Delta T$</td>
<td>change in temperature</td>
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<td>$\epsilon_p$</td>
<td>prestrain applied to a lamina</td>
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<td>$\epsilon_0$, $\epsilon_1$, $\epsilon_2$, $\epsilon_3$</td>
<td>von Karman strains</td>
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<td>$\kappa_0$, $\kappa_1$, $\kappa_2$, $\kappa_3$</td>
<td>curvatures of the midplane</td>
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References


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